OBERSEMINAR: FARRELL-JONES CONJECTURE

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The main goal of the is to understand the proof of the K-theoretic Farrell-Jones Conjecture for hyperbolic groups. All speakers are welcome to discuss their talk early on with me.

Introduction. (Arthur Bartels)
Introduce the Farrell-Jones Conjecture. Outline its proof for hyperbolic groups.

Algebraic K-theory. (Georg-Joachim Frenck)
The talk should give an introduction to algebraic K-theory. A possible starting point is the definition of K_0 and K_1 for additive categories. Later we will need that algebraic K-theory defines a functor from additive categories to spectra. Most of the argument later on only use the properties of K-theory listed in [4, Sec. 2.1]. On the other hand, the construction of the transfer uses (for higher K-theory) the framework of Waldhausen categories [16]. Interesting would also be a discussion of the Bass-Heller-Swan formula for the K-theory of $R[t, t^{-1}]$ [15, Thm. 3.2.22] and the vanishing of the Nil-terms for regular R [15, Ex. 3.2.25].

The Davis-Lück formulation of the Farrell-Jones Conjecture. (Florian Göppl)
The goal of this talk is the formulation of the Farrell-Jones Conjecture [12, Conj. 58] in the language of Davis and Lueck and the proof of the transitivity principle [12, Thm. 65]. Important is the connection to the classical assembly maps [12, Rem. 62] and the special case of torsion free groups and regular coefficients [12, Conj. 11]. The speaker should also discuss equivariant homology theories [12, Sec. 2.7.1-2.7.3], in particular the functor $\mathbf{K}_R$ [12, Eq. (2.37)] and the associated equivariant homology theory. The original reference for the Davis-Lück formulation of the Farrell-Jones Conjecture is [9].

Controlled topology I. (Grigori Avramidi)
This talk should give an introduction to controlled topology (or controlled algebra). One reference is [1, Sec. 2] where the thin h-cobordism theorem, geometric modules and the algebraic thin h-cobordism theorem are discussed. An alternative beautiful reference is [14] where a quick proof of topological invariance of Whitehead torsion is given.

Controlled topology II. (Johannes Ebert)
In this talk the more categorical view of controlled algebra is used. The speaker should sketch the construction of an equivariant homology theory via controlled topology [4, Sec. 3-5]. The assembly map can then be viewed as a forget control map. As a consequence of this construction one obtains the fiber of the assembly map as the K-theory of the obstruction category [6, Sec. 3]. The Farrell-Jones Conjecture translates now into triviality of the K-theory of the obstruction category. An important tool for understanding the obstruction category is stability [6, Sec. 7]. See [6, Sec. 4.4] for how stability enters the proof of the Farrell-Jones Conjecture.

Date: March 2013.

Finite VCyc**-amenability for actions.** (Lukas Buggisch)
This talk should introduce the axiomatic conditions from [6, Sec. 1]. The formulation of theses conditions is somewhat cumbersome; reformulations can be found in [1, Thm. A]. Later I started to use the term *finite* VCyc-*amenability* [1, Def. 0.1]. The talk should also discuss amenability for actions of groups on topological spaces [13, Def. 1.1]. Instructive is the example of the action of free groups on their boundary [13, Ex. 2.2]. An interesting fact the speaker could mention is that groups that admit some amenable action on a compact space are exact and are known to satisfy the Novikov conjecture [11]. Important for the rest of our seminar is the translation from long thin covers to contracting (alternatively almost equivariant) maps from [6, Sec. 5], see also [10, Prop. 4.6].

Transfer. (William Gollinger)
The goal of this talk is to construct the transfer map from [6, Sec. 6]. It might also be beneficial to recall how the transfer fits into the overall proof of the Farrell-Jones Conjecture [6, Sec. 4]. A simpler version of the transfer that addresses only K_1 (but not higher K-theory) can be found in [1, Sec. 4+6]. The speaker could choose to concentrate on this more pedestrian route. Time permitting it would also be interesting to discuss the L-theory transfer [5, Sec. 10].

Hyperbolic groups. (Divya Sharma/Michael Joachim)
The goal of this talk is to give some background about hyperbolic groups. Important are the Rips complex $P_d(G)$ [8, p. 468-470] of a hyperbolic group as a model for $E_{\mathrm{Fin}}G$, the boundary ∂G [8, Sec. III.H.3] of a hyperbolic group and the fact that the compactification $P_d(G) \cup \partial G$ of the Rips complex is an ANR [7, Thm. 1.2].

The action of a hyperbolic group on its boundary. (Robin Loose)
The goal of this talk is to proof that the action of a hyperbolic group on its boundary is finitely VCyc-amenable. It follows then quickly, that the same holds for the action on the compactification of the Rips complex. Axiomatic results discussed earlier imply then the Farrell-Jones Conjecture for hyperbolic groups. A short outline for surface groups is contained in [3, p.5+6]. The proof breaks naturally up in two steps. First define the coarse flow space for a hyperbolic group [2, p.4] and show that it admits long and thin covers [2, Thm. 1.1]. The result follows then using a suitable map from the Rips complex to the coarse flow space [2, p.5]. In the relative hyperbolic situation this is discussed in detail in [2, p.16-20]. The hyperbolic case follows a similar, but easier argument – all discussion of angles can be ignored.

The surgery exact sequence and the Borel Conjecture. (Michael Weiss)
The Farrell-Jones Conjecture is known to imply the Borel Conjecture in dimension ≥ 5. The goal of this talk is to outline how much surgery theory has to be invoked for this implication. (It is a lot.) A starting point could be the references given in the proof of [5, Prop. 0.3]. A more ambitious speaker could also discuss the implications of the Farrell-Jones Conjecture for Wall's realization question for Poincaré duality groups via Ranicki's total surgery obstruction.

References

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