THE ANALYTICAL ASSEMBLY MAP AND INDEX THEORY

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Abstract. In this talk I want to present a recent result of my own about a geometric interpretation of the abstract analytical assembly map as it appears in the Baum-Connes conjecture. The setup is as follows:

Let G be a countable discrete group, then the Baum-Connes conjecture predicts the so-called *analytical assembly map*

$$K_*^G(\underline{E}G) \xrightarrow{\mathcal{A}} K_*(C_r^*G)$$

to be an isomorphism of graded abelian groups between the G-equivariant analytical K-homology of the classifying space for proper G-actions and the K-theory of the reduced group C^*-algebra.

In the case where the group G is torsion-free there is an isomorphism $K_*^G(\underline{E}G) = K_*^G(EG) \cong K_*(BG)$. Moreover one can define a *Mishchenko-Fomenko index* map

$$K_*(BG) \xrightarrow{\mathrm{MF}} K_*(C_r^*G)$$

and it is natural to study the relation between these two resulting maps. This is precisely my result:

Theorem. *For every countable, discrete, torsion-free group G the diagram*

$$\begin{tikzcd} K_*(BG) \arrow[rr, "\mathrm{MF}"] & & K_*(C_r^*G) \\ K_*^G(EG) \arrow[u, "\cong"] \arrow[urr, bend right, "\mathcal{A}"'] & & \end{tikzcd}$$

is commutative.

In the first part of the talk I want to give a brief introduction to the Baum-Connes conjecture and construct the analytical assembly map and the Mishchenko-Fomenko map. Then I want to explain the ideas of the proof of the theorem. This proceeds in two steps, the first one is to show that both maps have the same "formal" properties. Having achieved this, we will then need to show that two certain elements that have been constructed in the first step are equal in a particular KK-group. In the last step we can use quite recent results by Buss–Echterhoff to prove the needed equality.