Loop groups and their representations

1. **Lie groups and Lie algebras: the classical theory.**
   Lie groups, Lie algebras, Cartan matrix, root decomposition, example: A_2 and B_2, Dynkin diagrams, classification
   References: [1], [3],[4],[6], and many more

2. **Central extensions of loop groups and loop algebras.**
   loop group, loop algebra, central extensions of loop algebra, central extension of loop group, universal extension of classical loop groups
   References: [4], [7],[8], [9, Chapter 4]

3. **Affine Lie algebras and Kac-Moody algebras**
   generalized Cartan matrix, affine Lie algebra, Kac-Moody algebra, extended Weyl group, affine Dynkin diagrams and their classification, affine algebras as central extensions of loop algebras
   References: [4], [7],[8]

4. **Representation theory: the classical theory.**
   finite dimensional representations of finite dimensional simple Lie algebras and their classification, highest weights, weight polytope, example: representations of $\mathfrak{s}l_2$, tensor product of representations, tensor product for representations of A_2 and B_2 respectively
   References: [1], [3],[5], [6], and many more

5. **Representation theory for the loop group.**
   Verma modules, construction of the highest weight representations, Weil modules, positive energy representations: construction and classification, decomposition series, Verlinde algebra
   References: [7],[8], [9]

6. **Kac character formula and applications.**
   Weyl character formula, generalized Casimir operator, Kac character formula, a proof thereof, applications, especially the example of $\hat{\mathfrak{s}l}_2$
   References: [7], [9]

7. **Fusion product via sheaves of coinvariants.**
   References: [2]

8. **Fusion product via von Neumann algebras.**
   References: [10]

References

[1] J. Frank Adams. *Lectures on Lie groups*. W. A. Benjamin, Inc., New York-Amsterdam, 1969.

[2] Bojko Bakalov and Alexander Kirillov, Jr. *Lectures on tensor categories and modular functors*, volume 21 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2001.

[3] Roger Carter, Graeme Segal, and Ian Macdonald. *Lectures on Lie groups and Lie algebras*, volume 32 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 1995. With a foreword by Martin Taylor.

[4] Jürgen Fuchs and Christoph Schweigert. *Symmetries, Lie algebras and representations*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, 1997. A graduate course for physicists.

[5] William Fulton and Joe Harris. *Representation theory*, volume 129 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991. A first course, Readings in Mathematics.

[6] James E. Humphreys. *Introduction to Lie algebras and representation theory*, volume 9 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1978. Second printing, revised.

[7] Victor G. Kac. *Infinite-dimensional Lie algebras*. Cambridge University Press, Cambridge, third edition, 1990.

[8] S. Kass, R. V. Moody, J. Patera, and R. Slansky. *Affine Lie algebras, weight multiplicities, and branching rules. Vols. 1, 2*, volume 9 of *Los Alamos Series in Basic and Applied Sciences*. University of California Press, Berkeley, CA, 1990.

[9] Andrew Pressley and Graeme Segal. *Loop groups*. Oxford Mathematical Monographs. The Clarendon Press Oxford University Press, New York, 1986. Oxford Science Publications.

[10] Antony Wassermann. Operator algebras and conformal field theory. III. Fusion of positive energy representations of $\mathrm{LSU}(N)$ using bounded operators. *Invent. Math.*, 133(3):467–538, 1998.