# EFFECTS OF TEACHING STUDENTS TO SOLVE OPEN MODELLING PROBLEMS ON UTILITY, INTRINSIC, AND ATTAINMENT VALUES

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*Task values are important for learning. However, prior research has indicated a lack of studies that have addressed students' task values in mathematics. In the following study (N = 293), we analyzed (1) the relationships between intrinsic, attainment, and utility values and (2) how teaching students to solve open modelling problems affects these values. Students in the experimental group were taught how to solve open modelling problems, whereas those in the control group were taught how to solve real-world problems with no missing information. Students reported their values before and after the intervention. The results revealed positive relationships between values plus a trend toward a positive effect of the intervention on utility value. We conclude that content-related interventions in modelling can improve motivational outcomes.*

## INTRODUCTION

Motivation comprises reasons for human actions (Middleton et al., 2016) and can be seen as a vehicle for human behavior. Prior research has analyzed how different aspects of motivation (e.g., values) are related to students' mathematical well-being (Hill & Seah, 2023) or the overall value of mathematics in the context of learning (Eccles & Wigfield, 2020). According to Eccles' expectancy-value theory of motivation (Eccles & Wigfield, 2020), students' mathematics values are very important for their achievement and educational choices. Values can be ascribed to different objects, such as the domain (e.g., mathematics), a topic (e.g., geometry), or a competency (e.g., modelling) (Schukajlow, Rakoczy, et al., 2023). As students who value mathematics take comprehensive mathematics courses in high school and choose mathematics as a study domain at universities, we addressed values in mathematics in this study.

One way to improve mathematical thinking is to teach students how to solve modelling problems. Solving modelling problems requires demanding processes through which information is transferred between the real world and mathematics (Niss & Blum, 2020). Openness is one important characteristic of modelling problems. However, the openness of modelling problems was found to be a source of various barriers that tend to arise in the solution process (Schukajlow, Krawitz, et al., 2023).

The aims of this study were (1) to analyze the relationships between three different types of values (i.e., attainment, intrinsic, and utility values; see below) and (2) to investigate how a teaching intervention aimed at improving students' ability to solve open modelling problems affects values in mathematics.



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## THEORETICAL BACKGROUND

### Task values as motivational outcomes

Task values are an important component of affective traits (Hannula, 2012). Values are stable motivational dispositions that can be related to learning and achievement. They can be distinguished from more temporary variable states, such as experiences of competence or autonomy in specific learning situations (Schukajlow, Rakoczy, et al., 2023). Task values indicate the personal importance of the tasks, such as the value of one's ability to solve a problem, the value of performing a calculation, or the value of making a drawing to solve a modelling problem. Expectancy-value theories assume that different learners ascribe different values to different tasks and thus, task values are subjective (Eccles & Wigfield, 2020). Furthermore, the extents to which one person values tasks vary across different tasks and different learning situations, indicating the situated nature of task values. In Eccles' expectancy-value theory, Eccles and colleagues proposed three key components of task values: attainment value, intrinsic/interest value, and utility/extrinsic value (Eccles & Wigfield, 2020). If a student sees mathematics as a part of their personality, ascribes mathematical achievement high personal relevance, and strongly identifies with mathematics, the student ascribes high attainment value to mathematics. The intrinsic value of mathematics concurs with enjoyment in solving mathematical problems and enjoyment in engaging in mathematical activities. The utility value of mathematics is reflected in the importance of mathematics for present or future plans, such as the importance of mathematics for learning in school, school grades, career, future work opportunities, or everyday life. Empirical studies have indicated that task values are positively related but distinct factors (Eccles & Wigfield, 2020). However, many studies have addressed the overall task value level by aggregating attainment, intrinsic, and utility values into one score or by using items that referred to the overall value of the presented mathematical problems (Böswald & Schukajlow, 2023; Rach, 2023). Very few studies have analyzed attainment, intrinsic, and utility values as distinct factors in mathematics and offered a more differentiated picture of the development of these motivational outcomes. One exception is a study by Gaspard et al. (2015). In this study, the authors found positive relationships (Pearson's correlations ranged from .50 to .71) between the attainment, intrinsic, and utility values of lower secondary school students. Furthermore, not many studies have analyzed how to improve values in mathematics. One potential way to improve students' values is to teach students to solve open modelling problems. But how can this type of problem be described?

### Open modelling problems

In the real world, many problems are open, and their solutions require assumptions to be made. Models of problem solving for open problems distinguish between the openness of the initial state, the openness of transformation, and the openness of the goal state. In modelling problems, the transformation is open because of the need to construct a mathematical model and select appropriate mathematical procedures to solve a problem. Depending on the type of open modelling problem, either the initial state or a goal state can be open (Schukajlow, Krawitz, et al., 2023). Problems with an open initial state do not include all the information needed for their solution. In problems with open goal states, the question is ambiguous, requiring interpretations about the quantity to be calculated to solve the problem. We know from prior research that dealing with openness is demanding for students and pre-service teachers (Galbraith & Stillman, 2001; Stylianides & Stylianides, 2023). In the current study, we focused on problems with an open initial state and a closed goal state. Analyses of cognitive demands that students face while solving modelling problems with an open initial state have indicated that noticing openness, identifying missing quantities, and making realistic assumptions about the missing quantities are essential prerequisites for processing the problems (Schukajlow, Krawitz, et al., 2023). For example, in the "Poster" problem, among other aspects, students should notice that the information about the diameter of the poster roll is missing, and they must make an assumption about its length. The goal state is closed in this task because the goal of the problem is to find out whether the poster will fit in the suitcase. To achieve this goal and solve the problem, students should compare the measurements of the poster roll with those of the suitcase. To do so, they need to calculate the diagonal of the interior of the suitcase.

Poster

Sandy is on vacation in Japan and would like to buy a movie poster there and roll it up to take home in her suitcase. However, she is unsure about whether it is possible to fit the poster in her suitcase. In a store, she finds a poster for 1075 yen. The poster is 105 cm long and 75 cm wide, and Sandy's suitcase is 40 cm long, 25 cm wide, and 60 cm high. When she rolls the longer side of the poster up, she gets a roll that is 75 cm long.

Can she transport the rolled-up poster in her suitcase?

Figure 1: The open modelling problem "Poster"

**Interventions to improve task values**

Findings from prior research on the effects of interventions that were aimed at increasing students' task values in different domains (e.g., mathematics) and specifically on the task values of solving modelling problems are mixed. Several studies have revealed that emphasizing the relevance of mathematics by prompting students to reflect on it affected task values and related outcomes (Rosenzweig et al., 2022; Schukajlow, Rakoczy, et al., 2023). Furthermore, the types of problems offered to students were shown to affect students' values. In a study of university students, future teachers reported higher intrinsic and utility values regarding profession-related tasks than for tasks that were not related to the teaching of mathematics in schools (Rach & Schukajlow, 2023). Because of the strong relationship between open modelling problems and the real world, students might value this type of problem more than problems that are not that strongly related to reality. However, contrary to this theoretical consideration, students and pre-service teachers did not assign higher overall value to solving open modelling problems compared with solving word or intra-mathematical problems (Böswald & Schukajlow, 2023). Two explanations for these findings are the higher perceived difficulty of open modelling problems and students' lack of familiarity with this type of problem. Teaching students how to deal with openness and how to solve open problems can increase the utility value of open problems and more generally the utility value of mathematics. As engagement in solving open modelling problems will promote the utility value of mathematics, and because of the positive relationships between utility value and the other values, we also expected an increase in attainment and intrinsic values.

## RESEARCH QUESTIONS AND EXPECTATIONS

This study was carried out in the framework of the Open Modelling Problems in Self-Regulated Teaching (OModA) project. In this project, we have been investigating how students solve open modelling problems and how teaching can support students' learning of open modelling problems and improve affective outcomes, including the extent to which students value mathematics (Schukajlow, Krawitz, et al., 2023).

Building on expectancy-value theory and the theory on modelling, the research questions in this study were:

RQ1: Are attainment, intrinsic, and utility values in mathematics positively related before the intervention?

On the basis of expectancy-value theory (Eccles & Wigfield, 2020) and prior empirical findings (Gaspard et al., 2015), we expected positive relationships between the three types of values.

RQ2: How does teaching students to solve open problems affect their attainment, intrinsic, and utility values?

We expected that teaching students how to solve open problems (compared with closed real-world problems) would increase students' feelings that mathematics is a part of their personality (attainment value), their enjoyment of mathematics (intrinsic value), and their perceptions that mathematics is useful (utility value).

## METHODOLOGY

One hundred eighty-five ninth graders from German middle and high track schools participated in this study (103 female; 14.5 years of age). Within each class, the students were randomly assigned to one of two groups. Students in the experimental group (EG) were taught how to solve open modelling problems, and students in the control group (CG) were taught how to solve closed real-world problems. Before and after the teaching intervention, students filled out questionnaires on values.

The EG learned how to deal with openness, missing quantities, and assumptions for solving open modelling problems. In the CG, students focused on how to solve problems with superfluous information. The problems in the CG did not require students to make assumptions about missing information in order to solve the problems. In both conditions, the teaching unit took 4 x 45 minutes. The study was conducted during regular classes in schools. Within the EG and the CG, students worked in smaller groups to solve the problems and then reflected on their solutions with the whole group (EG or CG) at the end of the class.

Pre-service teachers with bachelor's degrees in mathematics education served as teachers in this study. Before the study, they were given standardized training. To balance the effects of the instructor's personality on students, each teacher taught students in both EGs and CGs.

To assess values, we used well-evaluated Likert scales (ranging from 1 = not at all true to 5 = completely true) from prior studies. The scales on attainment, intrinsic, and utility values included 3 items each, and the scales' internal consistencies (Cronbach's Alpha) were at least acceptable (i.e., higher than .78). Sample items from the attainment, intrinsic, and utility value scales are: "It is important for me to be a person who is good at mathematics" (attainment value), "I like mathematics" (intrinsic value), "Mathematics is useful for my future life" (utility value).

To check the fidelity of the treatment, student assistants observed how the EGs and CGs were taught. Student assistants used a standardized observation questionnaire in which they were asked to note any deviations from the instructional manuals. In addition, we collected all student materials so that we could analyze the treatment fidelity. The analysis of the observational questionnaires and teaching materials indicated high treatment fidelity. In all classes, the teachers gave the tasks to the students in the same order in the EG and CG, and the teachers followed the teaching manual closely.

To address the research questions, we calculated Pearson correlations and conducted repeated-measures ANOVAs. In our statistical analysis, we included students who participated in the intervention (in the EG or CG) and filled out questionnaires at pretest and posttest ($N$ = 185). Students who missed the intervention or one of the tests were excluded from the analysis. The percentage of missing values ranged from 16.6% for utility value on the pretest to 21% for utility value on the posttest.

## RESULTS

The analysis of the correlations between the three values at pretest was in line with our expectations. Attainment value was positively related to intrinsic value ($r$ = .60, $p$ < .001) and utility value ($r$ = .54, $p$ < .001), and intrinsic value was positively related to utility value ($r$ = .51, $p$ < .001).

We conducted three repeated-measures ANOVAs with time as a within-subject factor (pretest, posttest) and treatment as a between-subject factor (EG, CG) and attainment value, intrinsic value, or utility value as dependent variables. The statistical analyses revealed mixed results. Contrary to our expectations, we did not find an effect of the interaction between the time and treatment factors on attainment value ($M_{EG,\ pre}$ $(SD)$ =

2.89 (0.93), $M_{CG,\ pre}\ (SD)$ = 2.82 (0.95); $M_{EG,\ post}\ (SD)$ = 2.77 (1.03), $M_{CG,\ post}\ (SD)$ = 2.59 (1.09); time*treatment: $F(1, 183) = 1125$, $p = .14$, $\eta^2 = 0.006$) or on intrinsic value ($M_{EG,\ pre}\ (SD)$ = 2.67 (1.03), $M_{CG,\ pre}\ (SD)$ = 2.63 (1.08); $M_{EG,\ post}\ (SD)$ = 2.43 (1.01), $M_{CG,\ post}\ (SD)$ = 2.50 (1.11); time*treatment: $F(1, 183) = 1093$, $p = .15$, $\eta^2 = 0.006$). These results indicate that teaching students how to solve open problems did not positively affect their attainment value or intrinsic value. The analysis of the effects on utility value indicated different results. In line with our expectations, the effect of the intervention on utility value was significant at the 10% level ($M_{EG,\ pre}\ (SD)$ = 3.23 (0.93), $M_{CG,\ pre}\ (SD)$ = 2.28 (0.94); $M_{EG,\ post}\ (SD)$ = 3.27 (0.90), $M_{CG,\ post}\ (SD)$ = 3.17 (1.01); time*treatment: $F(1, 183) = 1093$, $p = .09$, $\eta^2 = 0.009$). Thus, students who were taught how to solve open modelling problems tended to benefit more than students who were taught to solve closed real-world problems with respect to utility value.

## DISCUSSION

The analysis of the relationships between the three values in mathematics indicated positive relationships, supporting theoretical considerations from expectancy-value theory and results from prior studies (Eccles & Wigfield, 2020; Gaspard et al., 2015). One practical implication from this study is that it might be possible to primarily address one of the values in a teaching intervention, such as utility value, because the other values may then be affected through utility value.

To analyze how teaching students to solve open modelling problems affects students' motivation, we set up a randomized quasi-experimental study. Because of the significance of task values for achievement and educational choices, we selected students' values as motivational outcomes that can be affected by teaching. On the basis of expectancy-value theory (Eccles & Wigfield, 2020) and considerations from research on modelling (Niss & Blum, 2020; Schukajlow, Krawitz, et al., 2023), we hypothesized that if students learned how to solve open modelling problems (i.e., problems that are closely related to their real lives), they might subsequently value mathematics to a greater extent. After the teaching intervention, changes in students' attainment, intrinsic, and utility values were expected to be more beneficial for students in the EG that for students in the CG, who solved closed real-world problems. In line with our expectations, we found a positive trend toward an effect of teaching students to solve open modelling problems on utility value. This finding means that students who learned how to solve open problems tended to report higher utility value compared with students who learned how to solve closed real-world problems. This result supports theoretical considerations that, while learning how to solve open modelling problems, students perceive the relevance of mathematics for real life, and thus, they might understand how useful mathematics can be for their future lives or careers. This important result is in line with studies that have demonstrated the positive effects of interventions on the relevance of mathematics on values (Rosenzweig et al., 2022). Furthermore, this result supports the importance of the types of problems that students deal with in mathematics classes for the development of utility value. This consideration is hypothesized in expectancy-value theory and received empirical support in a prior study (Rach & Schukajlow, 2023). The theoretical implication of this study within the framework of expectancy-value theory is the indication that it may be possible to increase the fit between students' personal plans and doing mathematics by teaching open modelling problems. Future studies should clarify whether teaching open modelling problems fit into students' already existing plans (i.e., to use mathematics in the real world) or whether this intervention helped students develop such plans (i.e., helped them see the usefulness of mathematics in the real world).

Another important result of our study is the lack of effects of teaching students to solve open problems on attainment and intrinsic values in mathematics. One explanation might lie in how values in mathematics were assessed. Using modelling competence as an object of the values might reveal other results. The alignment between the object of the intervention and the measures may be a significant factor that affected the results of the study. In future studies, researchers should assess values with respect to open modelling problems and closed word problems (see an example of assessment in Böswald & Schukajlow, 2023). Another explanation might be that making mathematics personally important and improving enjoyment in mathematics might require more comprehensive instruction. Indeed, prior interventions have rarely targeted these values in the past (Rosenzweig et al., 2022), and it is very important to develop ideas about how teaching modelling problems can specifically improve attainment and intrinsic values. One way to increase intrinsic value might be to use contexts that refer to students' everyday lives (e.g., cities where students live), to adjust the context to students individual values (Bernacki & Walkington, 2018) or to ask them to pose their own problems (Voica et al., 2020). Furthermore, it is important to clarify in future studies what cognitive and motivational states during instruction mediate the effects of teaching students how to solve modelling problems on their task values.

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## References

Bernacki, M. L., & Walkington, C. (2018). The role of situational interest in personalized learning. *Journal of Educational Psychology*, *110*(6), 864–881. https://doi.org/10.1037/edu0000250

Böswald, V., & Schukajlow, S. (2023). I value the problem, but I don't think my students will: Preservice teachers' judgments of value and self-efficacy for modelling, word, and intramathematical problems. *ZDM - Mathematics Education*, *55*, 331–344. https://doi.org/10.1007/s11858-022-01412-z

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology*, *61*, Article 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Galbraith, P. L., & Stillman, G. (2001). Assumptions and context: Pursuing their role in modelling activity. In J. Matos, W. Blum, K. Houston, & S. Carreira (Eds.), *Modelling and Mathematics Education, Ictma 9: Applications in Science and Technology* (pp. 300–310). Horwood Publishing. https://doi.org/10.1533/9780857099655.5.300

Gaspard, H., Dicke, A.-L., Flunger, B., Brisson, B. M., Häfner, I., Nagengast, B., & Trautwein, U. (2015). Fostering adolescents' value beliefs for mathematics with a relevance intervention in the classroom. *Developmental psychology*, *51*(9), 1226–1240. https://doi.org/10.1037/dev0000028

Hannula, M. S. (2012). Looking at the third wave from the West: framing values within a broader scope of affective traits. *ZDM Mathematics Education*, *44*(1), 83–90. https://doi.org/10.1007/s11858-012-0410-5

Hill, J. L., & Seah, W. T. (2023). Student values and wellbeing in mathematics education: perspectives of Chinese primary students. *ZDM - Mathematics Education*, *55*, 385–398. https://doi.org/10.1007/s11858-022-01418-7

Middleton, J. A., Jansen, A., & Goldin, G. A. (2016). Motivation. In G. A. Goldin, M. S. Hannula, E. Heyd-Metzuyanim, A. Jansen, R. Kaasila, S. Lutovac, P. Di Martino, F. Morselli, J. A. Middleton, M. Pantziara, & Q. Zhang (Eds.), *Attitudes, Beliefs, Motivation, and Identity in Mathematics Education* (pp. 17–22). Springer.

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge.

Rach, S. (2023). Motivational states in an undergraduate mathematics course: relations between facets of individual interest, task values, basic needs, and effort. *ZDM – Mathematics Education*, *55*, 461–476. https://doi.org/10.1007/s11858-022-01406-x

Rach, S., & Schukajlow, S. (2023). Affecting task values, costs, and effort in university mathematics courses: the role of profession-related tasks on motivational and behavioral states. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-023-10413-7

Rosenzweig, E. Q., Wigfield, A., & Eccles, J. S. (2022). Beyond utility value interventions: The why, when, and how for next steps in expectancy-value intervention research. *Educational Psychologist*, *57*(1), 11–30. https://doi.org/10.1080/00461520.2021.1984242

Schukajlow, S., Krawitz, J., Kanefke, J., Blum, W., & Rakoczy, K. (2023). Open modelling problems: Cognitive barriers and instructional prompts. *Educational Studies in Mathematics*, *114*(3), 417–438. https://doi.org/10.1007/s10649-023-10265-6

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2023). Emotions and motivation in mathematics education: Where we are today and where we need to go. *ZDM - Mathematics Education*, *55*, 249–267. https://doi.org/10.1007/s11858-022-01463-2

Stylianides, G. J., & Stylianides, A. J. (2023). Promoting elements of mathematical knowledge for teaching related to the notion of assumptions. *Mathematical Thinking and Learning*, 1–29. https://doi.org/10.1080/10986065.2023.2172617

Voica, C., Singer, F. M., & Stan, E. (2020). How are motivation and self-efficacy interacting in problem-solving and problem-posing? *Educational Studies in Mathematics*, *105*(3), 487–517. https://doi.org/10.1007/s10649-020-10005-0