INTEREST AND PERFORMANCE IN SOLVING OPEN MODELLING PROBLEMS AND CLOSED REAL-WORLD PROBLEMS

Stanislaw Schukajlow[1], Janina Krawitz[1], Jonas Kanefke[1], Katrin Rakoczy[2]

[1]University of Münster, Germany, [2]University of Gießen, Germany

Modelling is an important part of mathematical learning. One characteristic feature of modelling problems is their openness. In this study, we investigated the relationship between interest and performance in solving open modelling problems and closed real-world problems. We used questionnaires and tests to assess the interest and performance of 143 ninth- and 10th-grade students at different achievement levels. We found that low-achieving students were more interested in solving open modelling problems than closed real-world problems. Also, prior individual interest in mathematics and performance were positively related to situational (task-specific) interest. These results contribute to interest theories by underlining the importance of types of real-world problems and achievement levels for situational interest.

INTRODUCTION

Modelling competencies are essential for mathematical learning. One important characteristic of modelling problems is their openness. In short, in our study, openness means that some important information is missing from the problem, and students must make assumptions about this information to solve the problem. Open problems can often be found in the real world, and thus, abilities to solve open modelling problems should be addressed in school. However, we do not know much about students' views on open modelling problems and their relationship to students' performance in solving this type of problem. We addressed students' interest as an important affective factor with high relevance for students' future educational choices (Hidi & Renninger, 2006) and examined differences in situational interest when solving open versus closed problems in high- and low-achieving students (i.e., students who attend middle- and low-track schools). We also analyzed how initial individual interest in mathematics and students' performance are related to situational interest in solving open modelling problems and closed real-world problems. We aimed to uncover the role of different kinds of mathematical problems (open modelling problems vs. closed real-world problems) in piquing students' situational interest. We seek to contribute to interest theories by clarifying how individual interest in mathematics and performance are related to situational interest in solving different types of mathematical problems.

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THEORETICAL BACKGROUND

Open modelling problems and closed real-world problems

To solve modelling problems, problem solvers must engage in the demanding transfer process between the real world and the mathematical world (Niss et al., 2007). Open modelling problems refer to problems with vague conditions. They do not include all the information needed to develop a solution, require problem solvers to make assumptions, and result in multiple solutions. Open modelling problems are examples of so-called ill-structured problems and rely on the model of ill-structured problem solving (Jonassen, 1997). Ill-structured problems are usually situated in a specific context in which one or more aspects of the problem situation are not specified, and the information needed to solve the problem is not completely provided in the problem. By contrast, well-structured problems provide all the information needed for a solution, and the problem solver just needs to select the relevant information from the task and link this information by using an appropriate mathematical procedure. In the past, a lot of research was carried out on closed real-world problems, whereas not much research on the affective and cognitive factors of modelling problems has focused on the openness of this type of problem.

Theoretical models of solution processes in mathematical modelling include, among other activities, understanding, structuring, simplifying, and idealizing a given situation (Blum & Leiss, 2007). Solving open problems requires problem solvers to notice missing information and make realistic assumptions about the situation described in the task and about the quantities that are missing (Krawitz et al., 2018). For example, while solving the Speaker problem (Figure 1), students need to notice that the information about the diameter of the speaker is missing and assume—by using the picture—that it might be one fourth of the height (about 5 cm).

Speaker

Maria bought the *Ultimate Ears BOOM* Speaker for 149.95 €. It has 360° sound with deep and precise bass. The speaker is 18.4 cm high.
Maria looks for a box with a cover for her speaker. On the web, she found a beautiful box. It is 14 cm wide, 10 cm high, and 14 cm deep.
Will the speaker fit in the box?

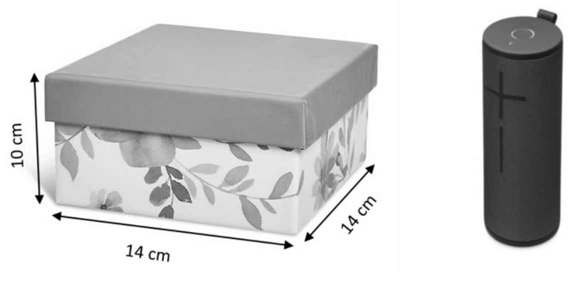

Figure 1: Open modelling problem "The Speaker".

Simplifying and idealizing are much easier when solving closed real-world problems. For example, after understanding the real-word problem Pyramid (Figure 2), students can directly construct the real model and the mathematical model, calculate the mathematical result, and interpret it to answer the question. In mathematics classrooms, closed real-world problems are a lot more common than open modelling problems.

Pyramid
The Cheops pyramid was built about 4,500 years before Christ, and it is the highest pyramid in Egypt. The blueprints show that the square base of the Cheops pyramid has a length of 230 m. The original lateral edge of the pyramid was originally 219 m long. Because the pyramid was used for a long time as a quarry, it is now only 138 m high. How many meters less is the Cheops pyramid now than it was before?

Figure 2: Closed real-world problem "Pyramid".

Interest and performance

Interest is defined as a relation between a person and an object (e.g., mathematical problems). Students with high interest engage with their object of interest over time (Hidi & Renninger, 2006). Models of interest development assume that unstable situational interest (state) develops into stable individual interest (trait), with individual interest strongly predicting situational interest (Hidi & Renninger, 2006). Individuals with more interest in mathematics can be expected to engage more often and more deeply in solving mathematical problems, consequently achieving higher performance. Many empirical studies have indeed found that the relation between individual interest and performance ranges from small to medium, depending on performance tests (Heinze et al., 2005). Situational interest assessed during problem solving in mathematics was found to be positively related to performance in solving problems and to initial individual interest in a prior study (Nuutila et al., 2020). Furthermore, students' initial individual interest in mathematics contributed to students' engagement and situational interest while solving the problem (Nuutila et al., 2020).

Motivational constructs, including interest, can address different objects (e.g., learning, mathematics, or modelling competencies) (Schukajlow et al., 2017). The strengths of relations between motivation-related measures (e.g., situational and individual interest) and performance depends on the domain and the types of problems. Researchers have found differences in situational interest between different domains (e.g., writing vs. mathematics) and within the domain of mathematics (e.g., interest in analytic vs. numerical reasoning) (Ainley et al., 2009; Nuutila et al., 2020). Prior research indicated that students' motivation (self-efficacy and task value) for solving open modelling problems was lower than for "dressed up" word problems (i.e., problems that do not require assumptions to be made, offer a model of the situation, and are related to closed real-world problems) (Krawitz & Schukajlow, 2018). This result contradicted the assumption that realistic problems are more motivating for students and was explained by the high difficulty of open modelling problems and students' lack of confidence in solving this type of problem. Further, researchers found that the relations between individual interest, performance, and situational interest depend on students' prerequisites and the type of task (Ainley et al., 2009; Nuutila et al., 2020). One explanation for this phenomenon is an alignment between the objects of initial individual interest: performance and situational interest. If the tasks offered in the classroom do not meet students' expectancies, the relation between initial interest on the one hand and performance and situational interest on the other might be weak. For open problems, if students with high mathematical interest solve an unfamiliar open problem that does not include all the information needed to solve it, the relations of individual interest and performance to situational interest might be weaker than for familiar closed real-world problems. Another important factor for the development of students' individual interest is students' prior performance. Problems that are too difficult or too easy for students might have a negative impact on situational interest. Prior research has rarely analyzed the relations between individual interest, situational interest, and performance for students with different levels of prior performance, even though it is important to determine the role of individual prerequisites (e.g., performance in this study) for the validity of theoretical assumptions and to draw practical implications from interest theories for the teaching of mathematics.

PRESENT STUDY, RESEARCH QUESTIONS, AND EXPECATIONS

This study was carried out within the framework of the project *O*ffene *Mod*ellierungs*a*ufgaben in einem selbständikeitsorientierten Unterricht (OModA), in English, Open Modelling Problems in Self-Regulated Teaching, which is aimed at investigating cognitive, strategic, and affective conditions for the teaching and learning of open modelling problems. Our research questions and expectations were:

RQ 1: Does students' situational interest in open modelling problems differ from their interest in closed real-world problems for both high- and low-achieving students? Because open problems are more realistic than closed problems (Blum & Leiss, 2007; Jonassen, 1997; Krawitz et al., 2018), we expected higher situational interest in solving open problems in both high- and low-achieving students.

RQ 2: Is students' initial individual interest in mathematics and performance related to situational interest in open modelling problems and closed real-world problems for high- and low-achieving students? On the basis of theories of the development of interest and motivation (Hidi & Renninger, 2006; Schukajlow et al., 2017), we expected initial individual interest and performance to be positively related to situational interest for both types of problems and in high- and low-achieving students. We had no clear expectations of differences between high- and low-achieving students.

METHOD

Sample, procedure, and measures

One hundred forty-three ninth graders (51% female; mean age = 15.66 years) participated in the study. The school system in the region of the study is organized such that, after attending primary schools, most students continue their education in mixed-track schools (Gesamtschule) or in high-track schools (Gymnasium). In order to capture students with different performance levels, we asked 76 students from a mixed-track school (called low achievers in this study) and 67 students from three high-track schools (called high achievers in this study) to participate voluntary in our study. Students filled out a questionnaire on individual interest in mathematics and took a performance test that included both open modelling problems and closed real-world problems in a mixed order. Immediately after solving each problem, students responded to the situational interest questionnaire.

Individual interest was assessed with a well-validated scale from a prior study ranging from 1 = not at all true to 5 = completely true (Frenzel et al., 2012) (six items, e.g., "I am interested in mathematics"). Internal consistency (Cronbach's α) was .84. Students' performances in solving open modelling problems and closed real-world problems included six problems of each of the two problem types. An example of an open modelling problem is the Speaker problem (Figure 1), and an example of a closed real-world problem is the Pyramid problem (Figure 2). To analyze performance in solving open modelling problems, students' solutions to these problems were scored 0 (wrong solution), 1 (no assumptions or unrealistic assumptions but otherwise accurate solution), or 2 (accurate solution under realistic assumptions). For performance in solving closed real-world problems, students were given a 0 for a wrong solution or a 1 for an accurate solution. The internal consistencies (Cronbach's α) of the instruments were .714 (open problems) and .711 (closed problems). Situational interest was assessed by asking students directly after solving each problem about their interest in solving the problem: "It was interesting to solve this problem" (1 = not at all true, 5 = completely true). We built a scale for situational interest in solving open modelling problems (six items) and closed real-world problems (six items) by calculating the mean across the respective types of problems. The internal consistencies were .78 (interest in solving open problems) and .83 (interest in solving closed problems).

We used ANOVAs, *t* tests, and regression analyses to address the research questions. Less than 5% of the students had missing values, when they skipped a questionnaire or did not solve any problems on the test. Students with missing values were excluded from the analyses.

RESULTS

Preliminary analyses confirmed differences between students in the high-achieving and low-achieving groups in their performances in solving open problems, $M(SD)_{high\text{-}ach} = .62(.33)$, $M(SD)_{low\text{-}ach} = .30(.28)$, $t(141) = 6.260$, $p < .001$, and closed problems, $M(\text{SD})_{high\text{-}ach} = .48(.28)$, $M(\text{SD})_{low\text{-}ach} = .21(.21)$, $t(141) = 6.459$, $p < .001$.

RQ 1 was about the differences in students' situational interest in solving open modelling problems and closed real-world problems (Table 1). A repeated-measures ANOVA with the factors type of problem (open vs. closed) and students' achievement (high vs. low) revealed no difference in interest regarding open versus closed problems across the whole sample, $F(1, 137) = .232$, $p = .37$, $\eta^2 = .006$. However, there was an interaction between type of problem and students' achievement, $F(1, 137) = 11.293$, p

< .001, $\eta^2 = .076$. Whereas high-achieving students had similar interest in solving open modelling problems and closed real-world problems with a slight tendency toward higher interest in closed problems, $t(66) = 1.825$, $p = .072$, Cohen's $d = 0.223$, low-achieving students were more interested in solving open modelling problems, $t(71) = 2.905$, $p = .003$, Cohen's $d = 0.342$.

Situational interest	High-achieving students *M* (*SD*)	Low-achieving students *M* (*SD*)
Interest in open modelling problems	2.98 (0.88)	3.01 (1.05)
Interest in closed real-world problems	3.12 (0.93)	2.79 (1.05)

Table 1: Means (standard deviations) for students' situational interest

RQ 2 was about the relations of prior individual interest, performance in solving problems, and situational interest (see the correlations in Table 2).

	Prior individual interest (1)	Performance open problems (2)	Interest open problems (3)	Performance closed problems (4)	Interest closed problems (5)
(1)	1	.235	.941**	.355**	.946**
(2)	.216	1	.158	.547**	.280*
(3)	.943**	.209	1	.291*	.783**
(4)	.271*	.772**	.235*	1	.372**
(5)	.946**	.208	.790**	.283*	1

* $p < .05$, two-tailed. ** $p < .01$, two-tailed.

Table 2: Pearson correlations for performance and interest in high-achieving students (above the diagonal) and low-achieving students (below the diagonal).

Prior individual interest in mathematics was strongly related to situational interest in solving open modelling problems and closed real-world problems. Students who were interested in mathematics were also interested in solving open and closed problems. The correlation was very high ($r > .9$) in high- and low-achieving students. Performance in solving open modelling problems was not related to situational interest in either achievement group, but performance in solving closed real-world problems was positively related to situational interest. Students who solved the real-world problems more accurately reported higher interest in this type of problem. When we included both individual interest in mathematics and performance as predictors of situational interest in solving real-world problems in a linear regression model, only individual interest remained statistically significant (high achievers: $\beta_{int} = .94$, $p < .001$, $\beta_{perf} = .026$, $p = .525$; low achievers: $\beta_{int} = .93$, $p < .001$, $\beta_{perf} = .041$, $p = .345$), indicating that individual interest was more important than performance for situational interest.

DISCUSSION

The goals of the present study were to identify the role of the type of problem (open modelling problems and closed real-world problems) for situational interest (i.e., task-specific interest) and to examine whether prior individual interest and performance were related to situational interest for high- and low-achieving students. In line with prior research (Krawitz & Schukajlow, 2018), the analysis revealed that high-achieving students reported similar interest in both types of problems with a slight tendency toward higher interest in closed problems. However, low-achieving students were more interested in solving open problems. This result is in line with theories of interest and modelling discussions, which assume that problems with a stronger connection to reality are more interesting for students, but why this was not the case for high-achieving students remains an open question. A possible explanation for the difference between high- and low-achieving students might be that low-achieving students did not notice that they needed to make assumptions to solve the open problems.

In line with interest theories (Hidi & Renninger, 2006), students' prior individual interest was found to be a strong predictor of situational interest. Interestingly, this finding held for traditional closed real-world problems and for less familiar open modelling problems. Students' performance was found to be related to situational interest for closed problems but not for open modelling problems. One reason for this result might be the differences in students' perceptions of the accuracy of their solutions for the two types of problems, which in turn might influence their situational interest. For example, some students might overlook the importance of the diameter of the speaker, calculate the diagonal of the box (see Figure 1), and assume that they developed the correct solution. The inaccurate perception of the correctness of a solution might decrease the relation between performance and situational interest in solving open problems in our study. A qualitative analysis of students' task processing and perceptions of the correctness of solutions to open problems is important to clarify this possibility. One important limitation of this study is that high- and low-achieving students can differ not only in their performance but also in other factors (e.g., learning materials distributed in the classroom) because they attend different types of schools.

The novel contribution of this study is that we addressed students' situational interest in open modelling problems. One theoretical implication of our study is the importance of individual interest in mathematics for the emergence of situational interest in different types of real-world problems and for students at different performance levels. Practical implications might be the possibility to evoke situational interest in low-achieving students by offering open modelling problems in the classroom.

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