EFFECTS OF TREATING MULTIPLE SOLUTIONS WHILE SOLVING MODELLING PROBLEMS ON STUDENTS' SELF-REGULATION, SELF-EFFICACY EXPECTATIONS AND VALUE

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In the project MultiMa (Multiple solutions for mathematics teaching oriented towards students' self-regulation) the effects of treating multiple solutions while solving modelling problems on students' learning are investigated. In the quasi-experimental study we report on the comparison of two groups of students. In one group modelling tasks, where the solutions do not demand making assumptions about missing data were treated. In another group students solved similar modelling problems, where different assumptions are possible, and students had to develop two and more different solutions each. About 120 9th graders from six middle track classes took part in this study for five lessons. Before and after a teaching unit students' self-regulation, self-efficacy expectations and value were tested.

INTRODUCTION

Although there are a number of results about some aspects of modelling, the influence of different treatments of modelling on student's self-perceptions is still an open question. One way to answer this question is an investigation of different learning environments and their effects on non-cognitive variables. In the recent study we focus on the improvement in self-regulation, self-efficacy and value in learning environments with und without a possibility to solve modelling problems in different ways.

THEORETICAL BACKGROUND

Self-regulation, self-efficacy expectations and value

Boekaerts (2002) distinguishes three main parts in the process of self-regulation: (1) students' orientation toward the attainment of their own goals, (2) the thoughts, feelings, and actions that can help them to attain these goals, and (3) working toward the attainment of their goals. Self-regulation is an essential aim of teaching preparing students for lifetime learning. Training in self-regulation and influences positive students` achievements and affect (Marcou & Lerman, 2007). According to Bandura (2003) self-efficacy expectations are "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments". Self-efficacy is connected with achievements in mathematics and with learning in general. Students with a high level of self-efficacy regulate their learning process more intensively and show better performances in mathematics (Malmivuori, 2006). "Value" characterizes the perceived importance attributed to objects, contents and actions (Eccles & Wigfield,

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2002). Values play an important role in theories of human motivation, which assume that students' motivation to learn is influenced by the importance attributed to learning and its objects. Students with high value beliefs in mathematics are described by teachers as better learners in the cognitive, metacognitive and motivational domains (Metallidou & Vlachou, 2010). The investigation of teaching methods for fostering students' self-regulation, self-efficacy expectations and value are important goals of mathematics education.

Multiple solutions while problem solving

Constructivist theories argue that developing different solutions and representations helps students acquire a multiple representation of the subject matter. Due to a multiple representation, students have a procedural flexibility in the respective domain and are able to solve unfamiliar problems. The crucial point while fostering a multiple representation in the classroom is a link between single representations and solutions. Thus teaching problem solving should stimulate the development of different solution methods, improve the connected mathematical knowledge and competencies as well as include a presentation of the individual solutions of students in the classroom (Leikin & Levav-Waynberg, 2007). Recently some experimental studies were carried out to identify the influence of treating multiple solutions on students' learning in mathematics (Rittle-Johnson & Star, 2007). Students that developed two solution methods for the same task outperform students that developed one solution at a time. Although investigation of emotions, attitudes, beliefs and other affective measures has been an important part of mathematics education research for decades (Zan, Brown, Evans, & Hannula, 2006), there is still a lack of studies that investigate the impact of different learning environments on students' self-perceptions. Moreover, we found no study that investigates the connection between developing multiple solutions and students' affect.

Multiple solutions and modelling

The important activities while modelling are simplifying a complex situation that is presented in the task, mathematizing and working mathematically to reach a mathematical result. While solving a modelling problem a problem solver can often choose among several possibilities to simplify a problem, to mathematize or to work mathematically. Different solution paths or methods can be chosen and sometimes there are different outcomes as result. To illustrate some of these activities we analyse the solution of the task "Parachuting", developed in the Framework of MultiMa-Project. First the problem solver has to understand the problem "Parachuting" and construct the situation model. Then the situation model should be simplified and structured. In order to do so, the problem solver has to make assumptions. The main assumptions while solving this problem are (see Schukajlow & Krug, in press):

- "the deviation remains constant at the different stages of the jump,
- the wind speed is light, middle or strong during the respective stages,

- the parachute opened e.g. at 1000 m above the ground."

Parachuting

When "parachuting", a plane takes jumpers to an altitude of about 4000 m. From there they jump out the plane. Before a jumper opens his parachute, he makes free fall of about 3000 m. At an altitude of about 1000 m the parachute opens and the sportsman glides to the landing place. While falling, the wind carries the jumper away. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand meters during free fall	Side deviation per thousand meters while gliding
Light	60 m	540 m
Middle	160 m	1440 m
Strong	340 m	3060 m

What distance does the parachutist cover during the entire jump?

Figure 1: Modelling task "Parachuting"

Next right-angled triangles should be identified, in these triangles the hypotenuses have to be calculated (e.g. with Pythagoras' theorem) and added in order to find a mathematical result. This result will be interpreted and validated. This analysis of solving the task "Parachuting" shows different ways to solve a modelling problem. Particularly changing assumptions lead to the different solutions and cause changes in the results of modelling. An important research question is: How does dealing with multiple solutions influence the modelling competency of students and their self-perceptions of mathematics? In the recent study we focus on the solutions that are developed as a result of the different assumptions students make and analyse the changes in self-perceptions of self-regulation, self-efficacy expectations and value. In the task "Parachuting" the assumptions are the wind velocity and the distance of the free falling stage among others.

Learning environments for treating modelling

In recent decades some key features of learning environments were identified as efficient classroom management, cognitive challenging activities, well-structured instructions and learning support of students (see e.g. Baumert et al., 2010). Special for treating modelling the sense making and modelling eliciting problems have to be used (Blum, 2011). The instructions in modelling were developed and evaluated in the Framework of the DISUM Project, where two teaching methods – "student oriented, self-regulated teaching method" and "teacher centered, directed instruction" were compared. Although the evaluation of this teaching unit showed positive results concerning students' achievements and affect (Schukajlow et al., in press), the students' progress is still disappointing from the normative point of view. One possibility to optimize this teaching unit is to integrate directive teaching elements like strong guidance of students at the beginning (Blum, 2011) and acquire the development of multiple solutions.

RESEARCH QUESTIONS

1. How many solutions do students develop in the group where multiple solutions were treated and are there differences in the number of developed solutions between this group and the group, where the development of only one solution was treated?
2. Are there differences in students' self-regulation, self-efficacy and value in mathematics due to the applied self-regulated teaching method of modelling problems?
3. Do students' self-regulation, self-efficacy and value differ according to the possibility to develop multiple solutions? In particularly, whether students that make different assumptions and develop multiple solutions while solving modelling problems report on other self-perceptions of self-regulation, self-efficacy and value than students in the group, where the development of only one solution was treated?

METHOD

Design and sample

138 German ninth graders (42.8% females; mean age= 15.2 years) were asked about their self-regulation, self-efficacy expectations and value before and after a five lesson period teaching unit (see Figure 2). Three schools with two middle track classes each took part on this study. Each of six classes was divided into two parts with the same number of students in each. The way the average achievements in the both parts did not differ and there was the approximately same ratio of males and females in each part. In the one part of each class multiple solution of modelling problems (group "multiple solutions") and in the other part one solution of modelling problems (group "one solution") were treated.

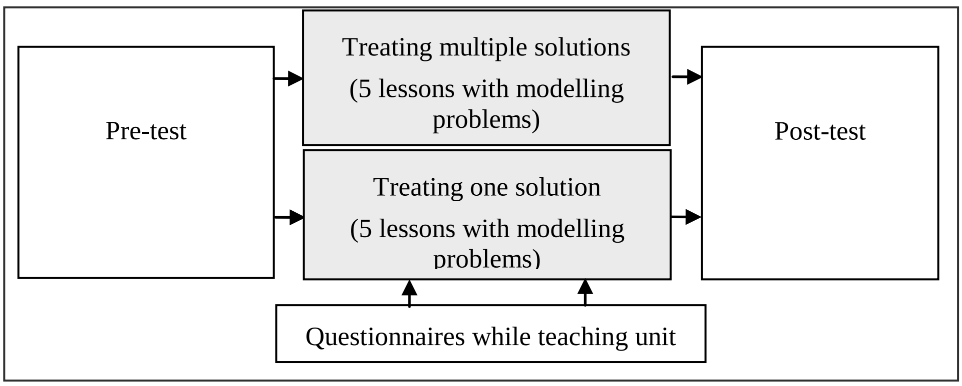

Figure 2: Overview of the study design

To implement the treating of modelling with and without multiple solutions two teaching scripts were developed. Four teachers that participated in this study received these scripts with all tasks to be treated and a detailed plan of the teaching unit. Further they were instructed about specific ways to promote modelling competency in both groups. Each teacher taught the same number of student groups in the group "multiple solutions" as in the group "one solution", so the influence of a teacher on students' learning did not differ between both groups. In each lesson one member of the research group was present to videotape and to observe the implementation of the treatment.

Treatment

The student-centered learning environment from DISUM Project was complemented by a directive instruction for the teaching unit used in the recent study. In both experimental groups the same methodical order was implemented. A teacher first demonstrates how modelling problems can be solved and multiple solutions can be developed. Students solve a modelling task according to a special kind of group work (alone, together and alone) and then discuss their solutions in the whole group in the classroom. A solution (or different solutions) of the first modelling task are presented by the students. The teacher has to summarise and to reflect on the key points of each group. In the group "multiple solutions" the teacher has to emphasise the development of different outcomes by estimating missing data. To foster the development of different solutions in one group and to prevent the development of more than one solution in the other group, two similar versions of each treated task were developed. The tasks in the group "multiple solutions" require the development of two solutions as, for example, in the task "Parachuting" (see Figure 1), where the question was "What distance does the parachutist cover during the entire jump? Find two possible solutions". In the group "one solution" students solved a version of this task where the main data needed to solve the task (the wind velocity and parachute altitude) were specified.

Measures

After every lesson the students were asked about a number of solutions they developed for the respective modelling problem. For example: "While solving the problem "Parachuting" I developed today (0: no solution; 1: one solution; 2: two or even more solutions)". Other students' self-perceptions were asked using a 5-point Likert scale (1=not at all true, 5=completely true) before and after a teaching unit (see Figure 2). The sample items were for self-regulation (6 items) "While learning mathematics I set my own goals which I would like to achieve", for self-efficacy (4 items) "I'm confident that I can understand the most difficult topics in mathematics" and for value (3 items) "I attach great importance to mathematics". All scales were adapted from the longitudinal PALMA study (Pekrun et al., 2007). Reliability values (Cronbach's Alpha) for self-regulation were .76 and .80, for self-efficacy .86 and .87 and for value .70 and .63 in pre- and post-test respectively.

RESULTS AND DISCUSSION

Research question 1

First we investigated how many solutions across all problems were developed in the group “multiple solutions”. The analysis of students’ answers shows that 4% of the students could not find any solution, 38% of the students developed one and 58% two or even more than two solutions. The majority of the students in the group “multiple solutions” developed two and more solutions (mean=1.55, standard deviation SD=0.39) as intended. In the group “one solution” students report on the development of two and more solutions less frequently (mean=1.14, SD=0.33). The analysis with t-Test ($T(138)=6.7$; $p<0.001$; effect size Cohen’s $d=1.16$) indicates that there are significant differences between the numbers of solutions that were developed in the respective groups. These results demonstrate that if you encourage students to find multiple solutions while solving modelling problems in the classroom, the majority of students really do it. In line with the intentions of the recent study there were significant differences in the numbers of developed solutions between students in the groups “multiple solutions” and “one solution”.

Research question 2

Teaching modelling problems for five lessons with a method used in the recent study improves students’ self-regulation, self-efficacy and value as an analysis with t-tests showed. All measured students’ self-perceptions are increased after the teaching modelling problems. The effect size Cohen’s d varies from small for self-regulation to medium for self-efficacy and value. We sum up that teaching modelling with a combination of directive guidance at the beginning of the teaching unit and group work in later parts improve students’ ability to regulate themselves, increase their self-efficacy expectation in mathematics and positively influence the students’ value of mathematics. However, one limitation is that the study design did not include a control group which is why it is not possible to exclude testing effects in interpreting these findings.

	Pre Mean(SD)	Post Mean (SD)	T(df)	p	Cohen’s d
self-regulation	3.62(.70)	3.79(.66)	2.8	<.01	.27
self-efficacy	3.23(.92)	3.57(.91)	5.2	<.01	.47
value	3.26(.90)	3.62(.85)	5.3	<.01	.49

Table 1. Students' self-regulation, self-efficacy and value at pre- and post-test

Research question 3

In order to investigate the impact of treating multiple solutions while solving modelling problems on measured self-perceptions of students we compared self-regulation, self-efficacy and value of both teaching environments in post-tests, taking into account their pre-test measures as covariate. The analysis with ANCOVA showed nearly significant differences between the groups “multiple solutions” and “one solution” in students’ self-regulation ($F(118)=3.5$, $p=.06$, effect size $(\eta)2=.03$). Students that were encouraged to develop multiple solutions while modelling most frequently report on regulation of their learning in mathematics after the teaching unit than students that have to develop only one solution. However, no differences between students in learning environments with and without treatment of multiple solutions were observed in self-efficacy expectations and value (Self-efficacy: $F(118)=0.6$, $p=.42$, $(\eta)^2=.01$; Value: $F(119)<0.1$, $p=.95$, $(\eta)^2<.01$).

	Treatment of multiple solutions		Treatment of one solution	
	Pre Mean(SD)	Post Mean (SD)	Pre Mean(SD)	Post Mean (SD)
self-regulation	3.69(.66)	3.93(.65)	3.52(.75)	3.64(74)
self-efficacy	3.29(.93)	3.64(.84)	3.14(.95)	3.49(.98)
value	3.31(.88)	3.66(.82)	3.15(.96)	3.57(.88)

Table 2. Students' self-regulation, self-efficacy and value before and after a teaching unit with and without treatment of multiple solutions

The results of the recent study show that a learning environment for treating modelling problems, where directive instruction and group work were combined, has positive influence on students’ self-regulation, self-efficacy and value. Moreover, the treatment of multiple solutions that are developed because of the different assumptions while solving modelling problems guide the majority of students to develop multiple solutions and increase their self-regulation. However, the treatment of multiple solutions has no effect on students’ self-efficacy and value. In further studies the effects of treating multiple solutions on affect and achievements need to be investigated.

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