DRAWING INSTRUCTIONS, STRATEGIC KNOWLEDGE, STRATEGY-BASED MOTIVATION, AND STUDENTS' USE OF DRAWINGS

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Although self-generated drawing is a powerful strategy in the domain of geometry, students lack spontaneous use of the drawing strategy. In the current study, we investigated instructional, cognitive, and motivational predictors of students' drawing use. We first assessed strategic knowledge about drawing and strategy-based motivation in 132 students in Grades 9 and 10. Then, students were randomly assigned to solve geometry modelling problems either with or without drawing instructions. Students with drawing instructions constructed more drawings than students without drawing instructions. Strategic knowledge about drawing, self-efficacy expectations, and perceived costs predicted drawing use while intramathematical abilities were controlled for. Utility value did not predict drawing use in the current study.

INTRODUCTION

Self-generated drawing is considered a powerful strategy for finding a solution to a geometry modelling problem. Although 13- to 15-year-old students are familiar with the strategy of self-generated drawing, many of them do not spontaneously use the drawing strategy (Uesaka et al., 2007). One way to increase students' use of drawings is to explicitly instruct them to make a drawing before solving a modelling problem. Previous research has indicated that a notable proportion of students still do not make a drawing even when instructed to do so (De Bock et al., 1998). Explanations for students' lack of drawing use include strategy-based cognitive and motivational factors. In the current study, we investigated how drawing instructions, strategic knowledge about drawing, and strategy-based motivation (self-efficacy expectations, utility value, and perceived costs) predict students' use of drawings to solve geometry modelling problems.

THEORETICAL BACKGROUND

The use of learner-generated drawings to solve geometry modelling problems

Past research has repeatedly shown that students experience diverse difficulties when solving modelling problems (e.g., Galbraith & Stillman, 2006). Modelling problems are ill-defined mathematical problems with a connection to reality that, amongst other functions, allow students to make realistic assumptions and apply different mathematical solution methods. An exemplary modelling problem is presented in Figure 1.

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Cable car

The municipality of Engelsberg needs to replace the steel rope of holding the cable car. One meter of the steel rope costs 9 €. How much will the new steel rope cost? The following data on the cable car are available:

Model:	Engelsberg cable car
Bottom station:	1,023 m above sea level
Top station:	1,605 m above sea level
Horizontal difference:	1,041 m
Transportation capacity:	585 passengers per hour
Driving speed:	9 m/s

Figure 1: Exemplary modelling problem *Cable car*.

One way to help students overcome difficulties in the modelling process is to instruct them to use powerful strategies, such as self-generated drawing (Galbraith & Stillman, 2006). The strategy of self-generated drawing describes the process of constructing a structurally analogous representation of the modelling problem on paper and to use it as a problem-solving aid (Van Meter & Firetto, 2013). From a theoretical perspective, the drawings that are used to solve modelling problems can be classified as situational or mathematical drawings (see Figure 2).

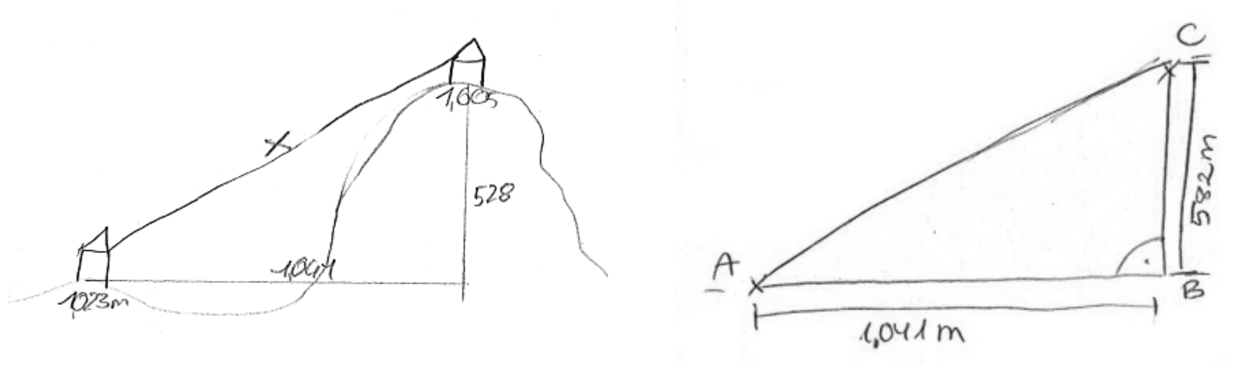

Figure 2: Exemplary situational (left) and mathematical (right) drawings by students.

The Cognitive Theory of Drawing Construction (Van Meter & Firetto, 2013) supports the assumption that making a drawing can help students work through the modelling process because it promotes the construction of mental models. A previous study confirmed that situational and mathematical drawings are powerful types of drawings that can help students solve a geometry modelling problem because students who made a more accurate situational or mathematical drawing solved the geometry modelling problem better than peers with a less accurate drawing (Rellensmann et al., in press). For example, a student can benefit from making a drawing for the modelling problem *Cable car* because the drawing may help them understand the relationship between the top and bottom stations or may help them figure out that the information they are looking for can be modelled as the hypotenuse of a right-angled triangle.

Drawing instructions

Although self-generated drawing is a powerful problem-solving strategy, previous research has shown that students often do not make drawings spontaneously (De Bock et al., 1998; Uesaka et al., 2007). For example, Uesaka and Manolo (2012) reported that 38% to 70% of students made spontaneous use of the drawing strategy to solve geometry word problems. One instructional approach that can be used to increase students' use of drawings is to explicitly ask them to make a drawing. Still, notable proportions of students do not follow such instructions (De Bock et al., 1998). Explanations for why students do not use drawings consist of deficient strategic knowledge about drawing (Rellensmann et al., in press) and a lack of strategy-based motivation (Uesaka & Manalo, 2012).

Strategic knowledge about drawing

Strategic knowledge about drawing (SKD) is specific strategic knowledge (Borkowski et al., 2000). It includes knowledge about the characteristics of an accurate drawing for solving a modelling problem, including the adequate representation of relevant objects and their relationships and complete labelling with relevant numbers (Rellensmann et al., 2020). According to the theoretical model proposed by Borkowski et al. (2000), SKD is an important precondition for the use of the drawing strategy. A recent study showed that improving students' SKD with strategy training resulted in more accurate drawings (Rellensmann et al., in press). Whether SKD predicts students' use of drawings has not yet been investigated.

Strategy-based motivation

Strategy-based motivation (SBM) is motivation that derives from the characteristics of strategies and their use. Referring to the expectancy-value theory of motivation (Eccles & Wigfield, 2020), SBM stems from specific manifestations of expectancies and value appraisals and can explain strategy-related decisions (e.g., the use of the drawing strategy). In the current study, we examined whether drawing-related SBM (self-efficacy expectations, utility value, and perceived costs) would predict students' use of drawings.

Regarding the drawing strategy, self-efficacy expectations comprise a student's confidence in being able to construct accurate drawings to solve modelling problems. A student with high self-efficacy expectations would give an affirmative answer to the question "Are you confident that you can make a very good drawing for any modelling problem?" Previous studies have found that self-efficacy expectations are positively related to drawing use (Uesaka et al., 2007) and drawing accuracy (Schukajlow et al., 2021). One explanation is that students who have more confidence in their abilities to generate accurate drawings set higher goals and engage in deeper learning processes compared with students who have less confidence in their drawing abilities. To date, no studies have investigated whether self-efficacy expectations affect drawing use.

Drawing-related value appraisals comprise utility value (Barron & Hulleman, 2015), that is, a student's belief that the activity of making a drawing is helpful for achieving their goals (e.g., solving the modelling problem). Previous empirical findings indicate that utility value predicts strategy use only when students have free choice of strategies: In studies on the spontaneous use of drawings, utility value positively predicted drawing use (Blomberg et al., 2020; Uesaka et al., 2007), whereas utility value did not predict drawing use when students were instructed to make a drawing (Schukajlow et al., 2021).

Another component in expectancy-value theories is the component of perceived costs (Eccles & Wigfield, 2020). Perceived costs of drawing comprise a student's belief about the amount of time and effort they need to invest to make a drawing. Previous research found negative relationships between the objective costs of drawing and spontaneous drawing use (Uesaka & Manalo, 2012) and negative relationships between perceived costs and drawing accuracy when students were instructed to make a drawing (Schukajlow et al., 2021). To date, it is an open question whether the perceived costs associated with making a drawing impede students' use of drawings.

RESEARCH QUESTION AND HYPOTHESES

In the current study, we investigated the following research question: Do drawing instructions, SKD, and SBM (self-efficacy, utility value, and perceived costs) predict students' use of drawings while mathematical abilities are controlled for? We expected that drawing instructions, SKD, self-efficacy expectations, and utility value would positively affect students' use of drawings, whereas perceived costs would negatively affect students' use of drawings.

METHOD

Procedure and participants

Participants were 132 students (45% female, 15–16 years old) in Grades 9 and 10 and in the middle achievement track of two German secondary schools. Students were randomly assigned to one of two groups: instructions to make a situational or mathematical drawing for each modelling problem ($n = 91$) and no instructions to make a drawing ($n = 41$). We aggregated students with situational and mathematical drawing instructions into the group *with drawing instructions* because the analyses did not reveal any differences between the groups with different drawing instructions. Data were collected on two different occasions to reduce the possibility that students in the control condition would be inadvertently prompted by the questionnaire to generate drawings. On the first data collection date, students worked on the test of intramathematical abilities, the strategic knowledge test about drawing, and the strategy-based motivation questionnaire. On the second data collection date, students were asked to solve eight modelling problems with or without drawing instructions.

Measuring instruments

Intramathematical abilities. To control for students' intramathematical abilities, we asked students to solve mathematical tasks without a connection to reality (10 items). For example, students were asked to set up an equation that fit a right-angled triangle or to solve a quadratic equation. Students' solutions were scored 0 (incorrect solution) or 1 (correct solution).

Strategic knowledge about drawing. The SKD scale (16 items) was developed and pilot tested in previous studies (Rellensmann et al., 2020). To solve an item from the SKD scale, students were asked to use a Likert scale to rate how helpful three situational drawings and three mathematical drawings were for solving a word problem. The three drawings that were provided differed in their accuracy. Students' evaluations of the three drawings were scored from 3 to 0 with respect to their accuracy.

Strategy-based motivation. To answer the strategy-based motivation questionnaire, students rated statements about themselves and their strategy-based motivation on a 5-point Likert scale. The items formed scales representing self-efficacy expectations (e.g., "I am confident that I can make a very good drawing for any word problem," 4 items), utility value (e.g., "I believe that it is important to make a drawing because making a drawing can help me solve a difficult word problem," 4 items), and costs (e.g., "I have to put forth a lot of effort to make a drawing for a difficult word problem," 3 items).

Drawing instructions. On the second data collection date, students worked on eight geometry modelling problems (see Figure 1). Students' group assignment was dummy coded: 0 (without drawing instructions) or 1 (with drawing instructions).

Drawing use. For each of the eight modelling problems, a student's use of a drawing was coded. When the student did not make a drawing, a code of 0 was given. When the student made a drawing, a code of 1 was given.

Interrater reliabilities (Fleiss' $\varkappa > .84$) and scale reliabilities (Cronbach's $\alpha > .64$) were satisfactory for all scales.

RESULTS

Correlations, means, and standard deviations for the investigated variables are presented in Figure 3. All correlations were in the expected directions, as SKD, self-efficacy expectations, and utility value were positively related to the use of drawings, and costs were negatively related to the use of drawings. Across the eight modelling problems, on average, 33% and 21% of the students made a drawing for a modelling problem in the groups with and without drawing instructions, respectively.

As we found notable correlations between the SBM components (e.g., $r = -.41$ between self-efficacy expectations and perseived costs), we computed multiple regression analyses with self-efficacy expectations, utility value, and perceived costs as simultanous predictors (Model 1) or separate predictors of students' use of drawings (Models 2a-c) (Figure 4).

	Intramathematical abilities (1)	Strategic knowledge about drawing (2)	Self-efficacy expectation (3)	Utility value (4)	Cost (5)	Use of drawings (6)
(1)	1	.10	.21*	-.10	-.23*	.31**
(2)		1	.05	-.04	-.13	.23*
(3)			1	.36**	-.41**	.30**
(4)				1	-.25**	.14
(5)					1	-.31**
(6)						1
M	0.26	1.97	3.25	2.62	2.65	0.27
SD	0.24	0.46	0.75	0.85	0.84	0.25

Note. * $p < .05$, ** $p < .01$, p two-tailed.

Figure 3: Correlations, means, and standard deviations for the investigated variables.

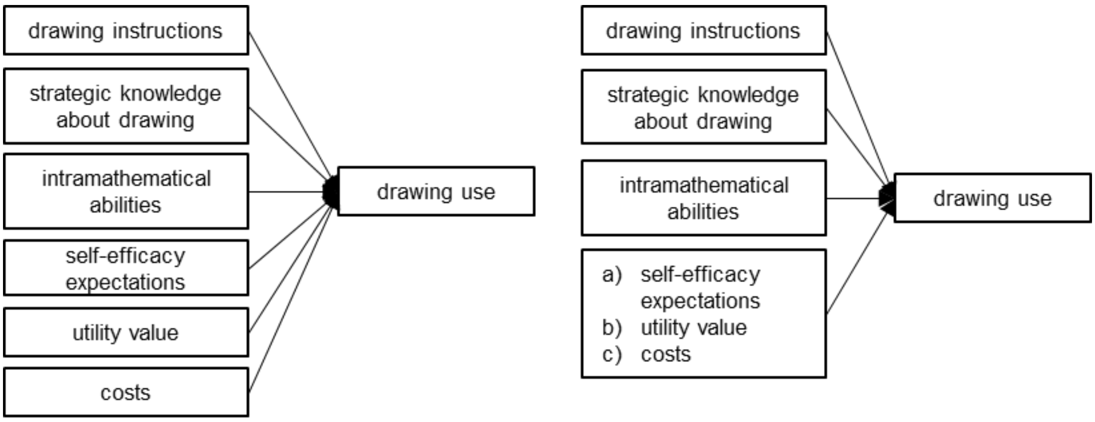

Figure 4: Model 1 with simultaneous SBM predictors (left) and Models 2a-c with separate SBM predictors (right).

In Model 1 with simultaneous predictors, we found that drawing instructions ($\beta = .19$, $p < .05$), SKD ($\beta = .19$, $p < .05$), and self-efficacy expectations ($\beta = .28$, $p < .01$) predicted students' use of drawings while intramathematical abilities were controlled for ($\beta = .17$, $p < .05$). Perceived costs ($\beta = -.11$, $p > .05$) and utility value ($\beta = .01$, $p > .05$) did not predict students' use of drawings.

In Models 2a, 2b, and 2c, we computed regression models with drawing instructions and SKD as predictors and intramathematical abilities as a covariate. We also entered self-efficacy expectations, utility value, and costs as separate predictors one at a time. In line with our hypotheses, self-efficacy expectations ($\beta = .31$, $p < .01$) and perceived costs ($\beta = -.22$, $p < .05$) were significant predictors of students' drawing use. Contrary to our hypothesis, utility value did not predict drawing use ($\beta = .04$, $p = .67$).

DISCUSSION

In line with previous research, we found that large proportions of students lacked spontaneous drawing use or did not follow drawing instructions to solve geometry modelling problems. This study contributes to previous research on drawing use, as we identified instructional, cognitive, and motivational predictors of drawing use while controlling for students' intramathematical abilities. First, we found that drawing instructions are an instructional means for overcoming students' lack of spontaneous drawing use. Thus, teachers might explicitly ask students to make a drawing to solve a modelling problem to give students more experience with the drawing strategy.

Second, as hypothesized in the model by Borkowski et al. (2000), we found that students with good SKD used drawings more often than students with lower SKD. This finding adds to previous research that showed that SKD is an important prerequisite for drawing accuracy (Rellensmann et al., 2020). Thus, teachers should aim to create opportunities for students to develop their SKD. Strategy training, which can increase students' SKD, involves instructional elements (e.g., comparing drawings of varying accuracy) that can be used to promote students' SKD (Rellensmann et al., in press).

Third, we found support for Borkowski et al.'s (2000) hypothesis that SBM affects strategy use. Our results extend Borkowski et al.'s (2000) model by indicating which components of SBM are particularly important for strategy use. As hypothesized, we found that students with high self-efficacy expectations used drawings more often than peers with lower self-efficacy expectations. Also, students who perceived drawing as too cost-intensive (i.e., taking too much time and effort) did not use drawings as much as students who perceived drawing to be less cost-intensive. Due to the strong correlation between self-efficacy expectations and costs, the effects of costs were no longer statistically significant when self-efficacy expectations were simultaneously considered in the regression model. Thus, self-efficacy expectations were found to be the stronger predictor of students' drawing use. Contrary to our expectations, utility value did not predict drawing use in the current study. One explanation is that utility value is powerful in educational settings that give students a choice between different strategies (e.g., Uesaka & Manalo, 2012). Thus, the current findings suggest that the promotion of SBM will help students make use of the drawing strategy. Further, prior research has demonstrated ways to enhance SBM in educational settings (Eccles & Wigfield, 2020). One way is for teachers to scaffold students' drawing construction (Zhang & Fiorella, 2019) to facilitate a mastery experience, thus enhancing students' self-efficacy expectations and reducing the perceived costs associated with drawing.

In the current study, we investigated relationships between strategy instructions, strategic knowledge, strategy-based motivation, and strategy use for self-generated drawing. Further research should confirm the relationships that were hypothesized in Borkowski et al.'s (2000) model for other strategies (e.g., backward or forward strategies).

Acknowledgments

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References

Barron, K. E., & Hulleman, C. S. (2015). Expectancy-value-cost model of motivation. In J. D. Wright (Ed.), *International Encyclopedia of the Social & Behavioral Sciences* (pp. 503–509). Elsevier Ltd.

Blomberg, J., Schukajlow, S., & Rellensmann, J. (2020). "Why don't you make a drawing?" Motivation and strategy use in mathematical modelling. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (pp. 46–54). PME.

Borkowski, J. G., Chan, L. K., & Muthukrishna, N. (2000). A process-oriented model of metacognition: Links between motivation and executive functioning. In G. Schraw & J. C. Impara (Eds.), *Issues in the measurement of metacognition* (pp. 1–41). University of Nebraska Press.

De Bock, D., Verschaffel, L., & Janssens, D. (1998). The predominance of the linear model in secondary school students' solutions of word problems involving length and area of similar plane figures. In *Educational Studies in Mathematics* (Vol. 35, Issue 1, pp. 65–83).

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology*, *61*, 101859.

Galbraith, P. L., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, *38*(2), 143–162.

Rellensmann, J., Schukajlow, S., Blomberg, J., & Leopold, C. (in press). Does strategic knowledge matter? Effects of strategic knowledge about drawing on students' modelling competencies in the domain of geometry. *Mathematical Thinking and Learning*.

Rellensmann, J., Schukajlow, S., & Leopold, C. (2020). Measuring and investigating strategic knowledge about drawing to solve geometry modelling problems. *ZDM*, *52*(1), 97–110. https://doi.org/10.1007/s11858-019-01085-1

Schukajlow, S., Blomberg, J., Rellensmann, J., & Leopold, C. (2021). The role of strategy-based motivation in mathematical problem solving: The case of learner-generated drawings. *Learning and Instruction*, 101561.

Uesaka, Y., & Manalo, E. (2012). Task-related factors that influence the spontaneous use of diagrams in math word problems. *Applied Cognitive Psychology*, *26*(2), 251–260.

Uesaka, Y., Manalo, E., & Ichikawa, S. (2007). What kinds of perceptions and daily learning behaviors promote students' use of diagrams in mathematics problem solving? *Learning and Instuction*, *17*, 322–335.

Van Meter, P., & Firetto, C. M. (2013). Cognitive model of drawing construction. Learning through the construction of drawings. In G. Schraw, M. T. McCrudden, & D. Robinson (Eds.), *Learning through visual displays* (pp. 247–280). Information Age Publishing.

Zhang, Q., & Fiorella, L. (2019). Role of generated and provided visuals in supporting learning from scientific text. *Contemporary Educational Psychology*, *59*, 101808. https://doi.org/10.1016/j.cedpsych.2019.101808