

ARE MATHEMATICAL PROBLEMS BORING? BOREDOM WHILE SOLVING PROBLEMS WITH AND WITHOUT A CONNECTION TO REALITY FROM STUDENTS' AND PRE-SERVICE TEACHERS' PERSPECTIVES

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In this study, we asked 100 ninth graders about their boredom while solving problems with and without a connection to reality. We additionally asked 163 pre-service teachers to judge students' task-specific boredom with respect to the same problems. Our results show that whereas students experienced the same level of boredom for problems with and without a connection to reality, pre-service teachers judged students' boredom as higher for problems without a connection to reality. Moreover, pre-service teachers' judgment accuracy of students' boredom was low for both problem types with huge variability among pre-service teachers.

INTRODUCTION

Emotions are important for mathematics learning and achievement (Hannula, Evans, Philippou, & Zan, 2004). In the mathematics classroom, mathematical tasks can induce emotions in students (McLeod, 1992), and it can be assumed that varying the types of tasks might induce different emotional reactions. For example, a student might enjoy working on a real-world problem but might be bored when solving a purely mathematical problem or vice versa. In order to enhance lesson quality, teachers should be aware of students' task-specific emotions as teachers select problems for their classes. Thus, teachers need to accurately judge students' task-specific emotions. The aim of this study was to investigate students' experiences of boredom as they solved problems with and without a connection to reality and the ability of pre-service teachers to judge students' task-specific boredom.

THEORETICAL BACKGROUND

Problems with and without a connection to reality

Mathematical problems can be divided into problems without a connection to reality and problems with a connection to reality, and the latter can be subdivided into modelling problems and “dressed up” word problems. Examples of all problem types are illustrated in Figure 1. The differences between the problem types arise from the cognitive processes that are necessary to solve the problems (Niss, Blum, & Galbraith, 2007). To solve a modelling problem, the student first has to construct a mental model of the realistic problem situation, which then has to be simplified, structured, and mathematized to construct a mathematical model of the problem. All cognitive processes in modelling are challenging for students, as structuring, for example, can include making assumptions about missing data. After the mathematical model is

constructed, mathematical methods can be applied to compute a mathematical result, which finally has to be interpreted and validated with regard to the real situation. In a “dressed up” word problem, the reality-related cognitive processes are less complex. A simplified situation model is already given and only has to be “undressed” to find the mathematical model. Validation of the real result is limited to checking the mathematics and does not include checking the hypothesized models. Modelling and “dressed up” word problems have in common that they require processes of transferring between reality and mathematics and vice versa. By contrast, in a problem without a connection to reality, the mathematical model is already given. Mathematical methods can be applied directly, and the mathematical result does not have to be interpreted in reality. All problem types are important for students’ learning (Schukajlow et al., 2012). For example, by solving problems without a connection to reality, students can practice mathematical procedures. Solving “dressed up” word problems can introduce students to modelling activities. And finally, by solving modelling problems, students can learn to apply their mathematical knowledge in reality.

Students’ experiences of boredom while solving mathematical problems

Mathematical problems can elicit emotional reactions in students (e.g. boredom; Hannula et al., 2004). Boredom is one of the most frequently experienced emotions in the mathematics classroom (Frenzel, Pekrun, & Goetz, 2007) and can negatively influence students’ thoughts, motivations, and achievements (Schukajlow, accepted; van Tilburg & Igou, 2012). The control-value-theory posits that students’ perceived competence and students’ value appraisals are important sources of students’ boredom (Pekrun, 2006). Students’ *perceived competence* is related to students’ ability to perform a task and depends on the difficulty of the task. As task difficulty can vary within problem types, the impact of task difficulty on students’ boredom should be taken into account in research on students’ task-specific boredom. Students’ *value appraisal* refers to the perceived valences and personal relevance of task activities and outcomes. Accordingly, boredom is elicited by a mathematical problem if the student perceives the activities of solving the problem to be meaningless (van Tilburg & Igou, 2012).

Value appraisals for problems with and without a connection to reality can have different sources. A student might attribute a high value to solving an intra-mathematical problem because he or she perceives that solving the mathematical problem is valuable in its own right (e.g. because the problems helps the student to understand a mathematical idea or to practice mathematical procedures). A student who attributes a high value to a problem with a connection to reality may perceive either solving the real problem or solving the inherent mathematical problem as a meaningful activity. Consequently, the experience of task-specific boredom can differ for problems with and without a connection to reality according to students’ task-specific value appraisal. In mathematics education, it seems to be a common belief that problems with a connection to reality can improve students’ affect in relation to

mathematics (Beswick, 2011). The underlying assumption is that real-world problems make students experience and value the usefulness of mathematics in real life. However, Beswick (2011) argues that there is a lack of evidence for the positive impact of real-world connections on students' affect. For example, previous research did not find a difference in students' enjoyment while solving problems with and without a connection to reality (Schukajlow et al., 2012). However, in other studies on this issue, the impact of task difficulty was not controlled for (Schukajlow & Krug, 2014).

Pre-service teachers' judgments of students' boredom

As solving problems is a central activity in mathematics classrooms (Hiebert et al., 2003), knowledge about students' boredom while solving mathematical problems is important for teaching quality. Teachers have to judge students' task-specific emotions in order to be aware of task-specific effects on students' boredom. The accuracy of judgments of students' cognitive and affective characteristics is regarded as a key aspect of teacher expertise. Previous studies have indicated a deficit in teachers' ability to judge students' affective characteristics (Givvin, Stipek, Salmon, & MacGyvers, 2001; Karing, Dörfler, & Artelt, 2013). As one example, Karing et al. (2013) reported low-to-medium correlations between teachers' judgments and lower secondary students' anxiety in mathematics. Pre-service teachers' ability to judge students' boredom is a concern in teacher education, but it has not been investigated yet.

Research questions

In this study, we examined three research questions:

1. Does students' task-specific boredom differ between problems with and without a connection to reality?
2. Do pre-service teachers' judgments of students' task-specific boredom differ between problems with and without a connection to reality?
3. Do pre-service teachers accurately judge students' task-specific boredom when students solve problems with and without a connection to reality?

METHOD

Procedure and participants

In this study, we asked 100 ninth-grade students (56% female) from two German comprehensive schools to indicate their task-specific enjoyment and boredom on a questionnaire administered after task processing. Students' mean age was $M = 15.97$ years ($SD = 0.93$). We additionally administered an adjusted questionnaire to ask 163 pre-service teachers (86% female) in their first university year to judge ninth graders' task-specific enjoyment and boredom when solving the problems. The pre-service teachers' mean age was $M = 21.01$ years ($SD = 2.51$).

Sample problems

We used eight problems with a connection to reality and four problems without a connection to reality. All problems could be solved by using the Pythagorean theorem.

Figure 1 shows sample problems for both problem types. The problems with a connection to reality could be subdivided into dressed up word problems (e.g. Table tennis) and modelling problems (e.g. Maypole).

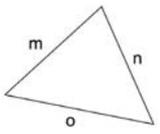
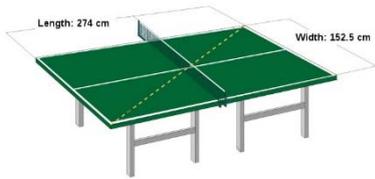
| | | |
|---|---|--|
| <p>Angle</p> <p>Where does the right angle have to be in the triangle (not drawn true to scale) so that the equation</p> $n^2 - o^2 = m^2$ <p>is satisfied?</p> <p>Draw the right angle into the triangle.</p>  | <p>Table tennis</p> <p>How long is the diagonal (dashed line) of a table tennis table?</p>  | <p>Maypole</p>  <p>Every year on Mayday in Bad Dinkelsdorf, there is a traditional dance around the maypole (a tree trunk approx. 8 m high). During the dance, the participants hold ribbons in their hands, and each ribbon is fixed to the top of the maypole. With these 15-m long ribbons, the participants dance around the maypole, and as the dance progresses, a beautiful pattern is produced on the stem (in the picture, such a pattern can already be seen at the top of the maypole stem).</p> <p>At what distance from the maypole do the dancers stand at the beginning of the dance (the ribbons are tightly stretched)?</p> |
|---|---|--|

Figure 1: Problem without a connection to reality (Angle) and problems with a connection to reality (Table tennis and Maypole)

Affect scales

To measure task-specific boredom, we adapted well-evaluated scales from previous studies (Schukajlow et al., 2012). In the questionnaires, each problem was followed by statements about students' affect.

In the students' questionnaire, the statement about boredom was "I was bored when working on this problem." Students rated the degree to which they agreed with the statements on a 5-point Likert scale (1=*not true at all*, 5=*completely true*).

In the pre-service teachers' questionnaire, the statement about students' enjoyment was "Students enjoy working on this problem," and the statement about students' boredom was "Students are bored when working on this problem." Pre-service teachers applied a 5-point Likert scale (1=*not true at all*, 5=*completely true*) to rate the degree to which the statements were true for ninth graders from a German comprehensive school.

Task difficulty

In order to exclude the confounding effect from task difficulty on task-specific boredom, we adjusted students' boredom values and pre-service teachers' judgments by the impact of task difficulty.

To adjust students' boredom values, we used students' task performance as an indicator of task difficulty. A code of 0 was given for an incorrect problem solution, and a code

of 1 was given for a correct problem solution. Inter-coder reliabilities for task performance were good ($\kappa > .86$).

To adjust pre-service teachers' judgments of students' boredom, we used pre-service teachers' perceptions of task difficulty, which were assessed in the questionnaire. Pre-service teachers used a 5-point Likert scale to rate the degree to which the statement "This task is too difficult for students" was true for ninth graders.

RESULTS

Preliminary results

In order to control for the impact of task difficulty on boredom, we computed adjusted boredom values. The adjusted values were only slightly different from the unadjusted values (Table 1). However, we used the adjusted values for our further analyses to control for the theoretically justified impact of task difficulty on boredom.

Table 1: Adjusted values for students' boredom and pre-service teachers' judgments

| Problem type | Students | | Pre-service teachers | |
|---------------------------------|------------------------|--|------------------------|--|
| | <i>M</i> (<i>SD</i>) | <i>M_{adj}</i> (<i>SD_{adj}</i>) | <i>M</i> (<i>SD</i>) | <i>M_{adj}</i> (<i>SD_{adj}</i>) |
| With a connection to reality | 2.46 (1.09) | 2.49 (1.08) | 2.59 (0.45) | 2.61 (0.44) |
| Without a connection to reality | 2.48 (1.12) | 2.46 (1.12) | 3.14 (0.73) | 3.11 (0.73) |

Students' boredom while solving problems with and without a connection to reality

Students' adjusted mean values on boredom were $M = 2.49$ ($SD = 1.08$) for problems with a connection to reality and $M = 2.46$ ($SD = 1.12$) for problems without a connection to reality (Table 1). Means and standard errors are graphically displayed in Figure 2. A t-test for dependent samples showed that the difference in students' adjusted task-specific boredom was statistically nonsignificant ($t(99) = 0.49, p > .05$). This means that students experienced the same level of boredom while solving problems with and without a connection to reality when the impact of task difficulty was controlled for.

Teachers' judgments of students' task-specific boredom

We also asked the pre-service teachers to judge the level of boredom that the students experienced while solving the same problems. When task difficulty was controlled for, pre-service teachers predicted a mean value of $M = 2.61$ ($SD = 0.44$) for problems with a connection to reality and a mean value of $M = 3.11$ ($SD = 0.73$) for problems without a connection to reality. A t-test for dependent samples revealed that the difference in pre-service teachers' judgments was statistically significant ($t(162) = -9.29, p < .05$) and that the effect size was large ($d = 0.73$). This means that pre-service teachers believe that students experience more boredom while solving intra-mathematical problems than while solving real-world problems.

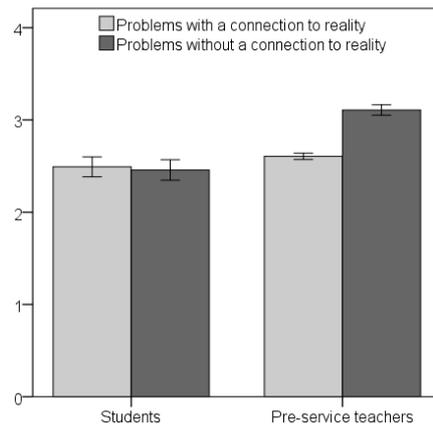


Figure 2: Means for students' boredom and pre-service teachers' judgments for problems with and without a connection to reality (Error bars represent standard errors)

Pre-service teachers' judgment accuracy

To assess pre-service teachers' accuracy in judging students' task-specific boredom, we estimated the level component and the rank component of judgment accuracy (Helmke & Schrader, 1987).

The level component of judgment accuracy relies on difference scores computed between students' boredom values and pre-service teachers' judgments and indicates whether pre-service teachers are able to accurately judge students' absolute levels of boredom. The mean difference scores indicated that pre-service teachers overrated students' boredom for problems with a connection to reality ($M = 0.09$, $SD = 0.40$) and problems without a connection to reality ($M = 0.64$, $SD = 0.74$). Single-sample t-tests showed that difference scores for problems with and without a connection to reality differed significantly from a value of 0, which stands for accurate judgments ($t(162) = 2.88$, $p < .01$, $d = 0.23$ and $t(162) = 11.12$, $p < .01$, $d = 0.86$, respectively).

The rank component of judgment accuracy indicates whether pre-service teachers are able to rank problems according to the level of boredom that the problems induce in students. For students' boredom, the mean correlation was $r = .02$ ($SD = .37$) for problems with a connection to reality and $r = .02$ ($SD = .70$) for problems without a connection to reality. Near-zero correlations and a huge range of correlations indicated that pre-service teachers have trouble judging students' task-specific boredom and that the ability to make accurate judgments differs greatly among pre-service teachers.

DISCUSSION

In this study, we found that students experience the same level of boredom while solving problems with and without a connection to reality when the difficulty of the assessed problems was taken into account. According to the hypothesized relation between feelings of boredom and the subjective values of activities in the control-value-theory (Pekrun, 2006), it can be assumed that students perceive intra-mathematical problems and real-world problems as equally meaningful. This means

that students perceive that solving an intra-mathematical problem (e.g. to understand a mathematical idea) is a valuable activity in its own right and that its value is not necessarily extended by a real-world connection. This result is in line with previous findings on students' task-specific enjoyment (Schukajlow et al., 2012).

In our study, pre-service teachers predicted that students would experience more boredom while solving problems without a connection to reality. This finding might indicate that pre-service teachers believe that students place more value on the use of mathematics to solve problems in the real world than they do on intra-mathematical problem solving—a commonly articulated argument in favor of real-world problems (Beswick, 2011). However, our study shows that students do not perceive intra-mathematical problem solving as particularly boring.

In line with previous research (Karing et al., 2013), our findings on pre-service teachers' judgment accuracy indicate that pre-service teachers have trouble judging students' boredom. Pre-service teachers overrated students' boredom for both problem types and were not able to rank problems according to the level of boredom that students experience while solving the problems. Moreover, our results showed huge variability in judgment accuracy among pre-service teachers. The deficit in pre-service teachers' ability to judge students' emotions should be addressed in teacher education and classroom practice. One method that can be used to improve teachers' knowledge about students is student feedback (Hattie, 2013). Regularly asking students to give feedback on their emotions can help teachers improve their judgment accuracy and enable them to match their teaching to students' learning conditions, which can improve learning.

Limitations and future directions

In this study, we distinguished between problems with and without a connection to reality. However, problems with a connection to reality can be subdivided into modelling problems and dressed up word problems. Although Schukajlow et al. (2012) did not find differences in students' boredom for modelling and dressed up word problems, it remains an open question whether students' experiences of boredom vary for different types of real-world problems when the impact of task difficulty is controlled for.

Conclusion

Are mathematical problems boring? Our results show that students and pre-service teachers answer this question differently. Whereas students report the same level of boredom while solving problems with and without a connection to reality, pre-service teachers judge students' boredom as higher for problems without a connection to reality. This result indicates a deficit in pre-service teachers' ability to judge students' task-specific boredom, which could also be seen in pre-service teachers' trouble in ranking problems according to students' task-specific boredom.

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