THE ROLES PRESERVICE TEACHERS ADOPT IN MODELLING-RELATED PROBLEM POSING

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Posing problems that are based on given real-world situations is important for teaching mathematical modelling. However, little is known about the posing process and the corresponding roles that teachers play. To help fill this gap, the current study examined the roles that preservice teachers adopt when they pose problems based on given real-world situations. We analyzed data from seven preservice teachers who posed problems based on a given real-world situation and identified three different roles preservice teachers tend to adopt when they pose a problem: protagonist, teacher, and problem solver. Further, we describe the domains that are addressed in these roles and affect teachers' decision-making when posing problems. Implications for how to teach (preservice) teachers to pose real-world problems are discussed.

INTRODUCTION

In order to teach mathematical modelling, teachers need to be able to pose adequate real-world problems (Blum, 2015; Borromeo Ferri, 2018; Greefrath et al., 2022). Modelling-related problem posing begins with the prompt to pose a mathematical problem based on a given real-world situation and results in a problem that can be solved afterwards (Hartmann et al., 2022). Despite the ongoing emphasis on approaches for teaching mathematical modelling, little is known about the process of modelling-related problem posing and especially the roles that have to be anticipated within this process (Geiger et al., 2021). To support (preservice) teachers in posing problems as they teach mathematical modelling, the aim of the present study is to identify and conceptualize the roles that preservice teachers adopt when posing problems based on given real-world situations, including the domains that are related to the decision-making processes that occur while posing a problem.

THEORETICAL BACKGROUND

Mathematical modelling (i.e., solving real-world problems with the help of mathematics) is one of the key competencies in mathematics education and is included in curricula all over the world (National Council of Teachers of Mathematics [NCTM], 2000; Niss & Blum, 2020). In order to teach mathematical modelling, teachers must have modelling-specific content knowledge, including knowledge about interventions, problems, and perspectives (Greefrath et al., 2022). Modelling-specific content knowledge includes knowledge about how to pose adequate problems (Borromeo Ferri, 2018).

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The term problem posing subsumes many different processes. In the context of mathematics education, it refers to problem development that is triggered by a stimulus and results in a mathematical problem that can be solved afterwards (Baumanns & Rott, 2022; Silver, 1994). Different situations and prompts can serve as stimuli (Cai et al., 2022). In order to pose problems based on given real-world situations, the problem poser first has to understand and explore the given situation. Then a problem has to be generated. The self-generated problem has to be evaluated with regard to individual criteria (e.g., solvability, adequacy for a specific learning group or adequacy in the given situation), and then possible solution steps can be planned (Hartmann et al., 2022). Hence, in order to pose modelling-related problems, demanding translation processes between the extramathematical and mathematical domains are needed, and many decisions have to be made.

To support (preservice) teachers in posing real-world problems, Galbraith (2006) described principles for the posing process. These principles serve as structural components that can scaffold the posing process and provide guidance on the important characteristics of the posed problem. One of the principles is that the posed problem should be connected to students' lives. Further, it should be possible to translate the posed problem into a mathematical problem. The solution to the posed problem should be feasible for the students and should require the application of modelling-specific activities (e.g., simplifying and structuring, interpreting, validating). Lastly, from a didactic perspective, it should be possible to divide the problem into subproblems in order to scaffold the solution process. The principles reveal that, when posing real-world problems, the extramathematical domain, the mathematical domain, and the didactic domain have to be kept in mind. To take these domains into consideration, the problem poser might adopt different roles that go along with focusing on important aspects of these domains. However, systematic research on the roles that (preservice) teachers adopt when posing problems based on given real-world situations has thus far been missing.

RESEARCH QUESTIONS

To help fill this gap, the overarching goal of this study was to identify and conceptualize the roles that preservice teachers adopt in modelling-related problem posing based on empirical data. To do so, we asked the following research questions:

a) What roles do preservice teachers adopt when they are instructed to pose mathematical problems that are based on given real-world situations, and how can these roles be described?

b) Which of the abovementioned domains are related to the decision-making processes applied in each role, and which aspects of the domains are taken into account?

METHOD

Sample

Data were collected from seven preservice teachers from a large German university (four women, three men) between 20 and 26 years of age. All of them participated in the program for future secondary school teachers: five of them for secondary and high schools (Grades 5-12/13) and two of them for secondary schools (Grades 5-10). Six of them reported having previous experience posing mathematical problems for students.

Procedure and Instruments

The analysis was based on data from a prior study for identifying the cognitive processes involved in modelling-related problem posing (Hartmann et al., 2022). For this purpose, the preservice teachers were instructed to pose a problem that was based on a given real-world situation while thinking aloud and to solve them subsequently. An example of a real-world situation that was presented is given in Figure 1.

Cable Car

For more than 90 years, the *Nebelhorn* cable car has taken numerous guests up into the heights. Now it can go into well-earned retirement. Beginning in the summer of 2021, a new cable car will transport enthusiastic outdoor fans up the Nebelhorn mountain. The aim of the project is to avoid long waiting times, provide seated transportation with an optimal view from every seat, and increase the cable car's carrying capacity.

Technical data of the old cable car:

Model:	Großkabinen-Pendelbahn
Weight empty cabin:	1600 kg
Weight full cabin:	3900 kg
Height valley station:	1933 m
Height top station:	2214.2 m
Horizontal difference:	905.77 m
Speed:	8 m/s
Carrying capacity:	500 people/hr
Power unit:	120 PS

Figure 1: The real-world situation of the cable car

The basis of the current analysis were the videos and the corresponding transcripts of the preservice teachers' posing processes as they posed problems that were based on the given real-world situation of the cable car (see Figure 1).

Data Analysis

In order to uncover the roles that the preservice teachers adopted, we used Mayring's (2015) content analysis. In a first step, we paraphrased the transcripts with regard to content-bearing semantic elements (sequences). In a second step, we developed the coding scheme inductively by using a collaborative process. This involved going through several rounds. In the first round, we reviewed the posing processes, began with an open coding, and created memos. Through an analytic process, we then developed codes. By using subsumption, we extended and refined the codes. For each of the sequences, we decided whether they fit into one of the existing codes or whether a new code had to be developed. In the next round, we gathered the sequences that were coded with the same code and discussed the descriptions of the codes collaboratively. Then we coded the rest of the data. This involved making several adjustments to the descriptions. Further, we found that a lot of sequences could not be clearly assigned to just one code. Hence, we decided that a sequence could be assigned to more than one code simultaneously. On the basis of the analytic process, the roles of the preservice teachers evolved from the data. The analysis resulted in a coding scheme that was used to analyze the data (see the Research Findings section). In a final step, we summarized the posing process for each preservice teacher with regard to the roles they adopted.

RESEARCH FINDINGS

On the basis of the preservice teachers' posing processes, we were able to develop a coding scheme that is presented in Table 1. The coding scheme includes the identified roles, the descriptions of the roles, and the domains related to decision-making.

Role	Description	Domain
Protagonist	The problem poser has adopted the role of a protagonist who wants to pose a relevant problem for themself or the given situation.	The extramathematical domain is in the foreground, and decisions are made on the basis of the description of the given real-world situation.
Problem Solver	The problem poser has adopted the role of a problem solver who wants to pose a mathematical problem with an interesting mathematical solution.	The mathematical domain is in the foreground, and decisions are made on the basis of the mathematics (i.e., mathematical operations and structures) that can be used to solve the problem.
Teacher	The problem poser has adopted the role of a teacher who wants to pose a suitable problem for their students.	The didactic domain is in the foreground, and decisions are made on the basis of the potential learning group and their knowledge.

Table 1: Roles involved in modelling-related problem posing

In the posing processes, we were able to identify three different roles (i.e., *protagonist*, *teacher*, and *problem solver*) the preservice teachers adopted when posing problems based on given real-world situations. As evidence that the preservice teachers actually integrated the roles into their modelling-related problem posing, we present Max's posing process, including the roles he adopted and the domains that he focused on for decision-making in the roles.

Max begins by reading about the given situation in the role of a problem solver by asking about what the mathematical context is.

> That's all well and good, (laughs) but what is the actual mathematical context now?

Then he identifies the information the question should refer to in the role of a problem solver and of a protagonist.

> Okay and now, my question should probably also refer to the new cable car, which will now be built beginning in summer 2021.

From the information he wants to focus on, he goes on to determine the mathematical content in the role of a problem solver.

> And there again I have to ask myself what mathematical concept I want to incorporate, so to speak.

He goes back to the real-world situation by making sense of the given situation in the role of a protagonist but also in the role of a problem solver.

> So, it's also about changing something about the old data now, so that, um, the waiting times can also be reduced and, also the optimal view is made possible, for example, and the conveying capacity is increased. Okay.

After making sense of the given situation, he continues in the roles of a protagonist and a problem solver by choosing information that he wants to work with mathematically.

> … because there is now a lot of information about the old cable car, I could also theoretically work with it and formulate a question that now refers to the old one. That makes more sense to me right now.

Then he switches to the role of a teacher by looking for mathematics that can be practiced with the self-generated problem.

> You could, for example, again theoretically use the Pythagorean theorem, and in principle, check or practice it in this task. Apply. The problem could be, for example, that the length of the actual route of this, um, railway, i.e., that the railway has to cover, is not specified here at all.

The upcoming problem is then evaluated in the role of a teacher by anticipating what mathematical knowledge students need to have in order to solve the problem and think about an appropriate formulation for the self-generated problem.

> Above all, it is also used to test… It is also used to… Or the pupils are also required to convert units because this also has to be switched here in any case between seconds and hours and so on. Now I'm wondering how my question should be formulated.

After posing a problem, Max checks the solution to his self-generated problem in the role of a problem solver.

> The beauty of the task, I think, is that you have to work through the different steps bit by bit in order to solve it [...]. So, we can't determine from the transport capacity the number of people per – that fit into a gondola. Exactly. We have to determine it from different things together. So, we have to go through different steps to solve those.

Figure 2 presents a schematic overview of Max's posing process with regard to the roles he adopted over time.

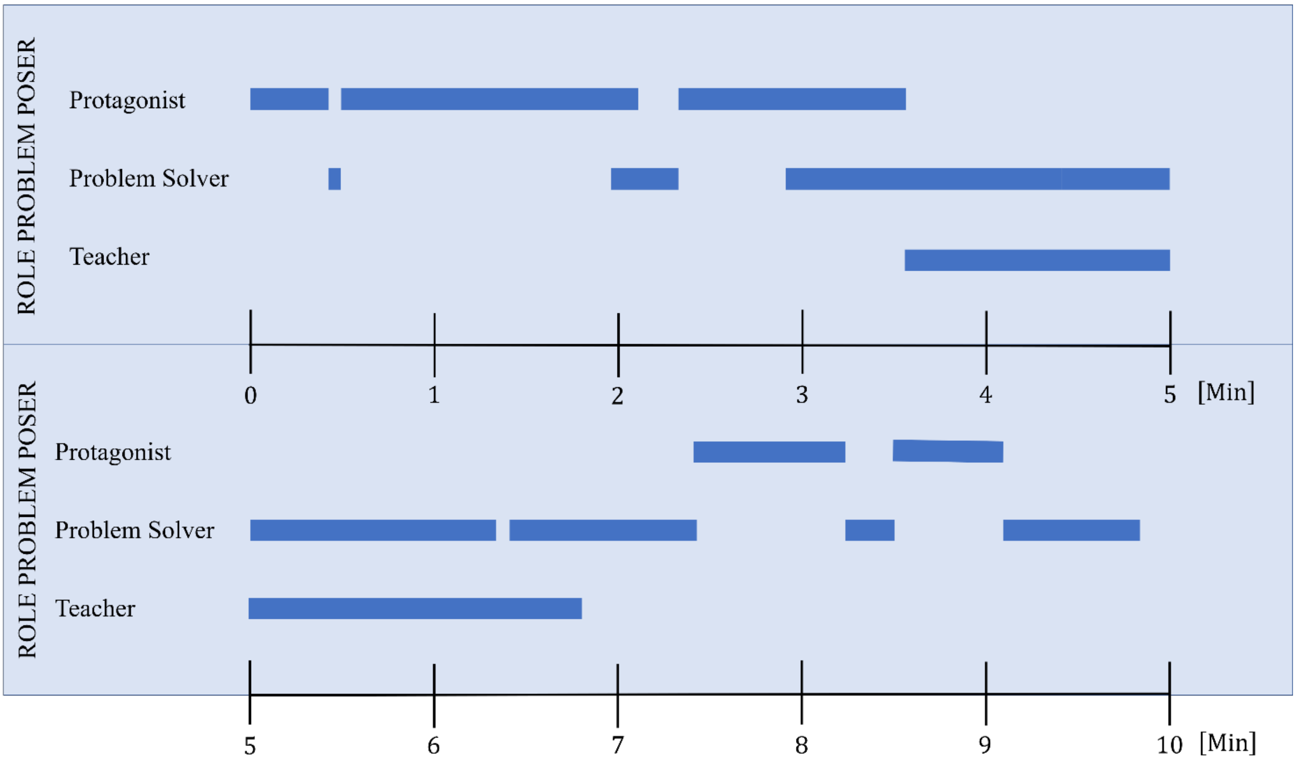

Figure 2: Overview of Max's posing process

Max's posing process (see Figure 2) reveals that the roles do not necessarily occur separate from each other. Rather, a duality of the roles can be observed and especially the role of the problem solver goes along with the other roles. Different profiles for the roles could be observed in the posing processes of the other preservice teachers.

DISCUSSION

In the modelling-related problem posing processes, three different roles could be identified. The preservice teachers adopted the roles of a *protagonist*, a *problem solver*, and a *teacher*. Adopting these roles seems to be relevant for making appropriate decisions when posing problems that are based on given real-world situations. Decisions are made by focusing on aspects of the extramathematical domain (in the role of a protagonist), the mathematical domain (in the role of a problem solver), and the didactic domain (in the role of a teacher). Galbraith's (2006) principles reveal that it is important to take these domains into account when posing real-world problems. Therefore, it is possible that instructing (preservice) teachers to put themselves in a specific role might encourage them to consider important aspects of the extramathematical, mathematical, and didactic domains and might therefore enhance the posing process. Future research has to reveal whether instructing (preservice) teachers to put themselves in a specific role has a positive effect on the posing process.

Further, the results show that the roles do not necessarily occur separately from each other. Rather, a duality of the roles could be observed in the data. Especially the role of the problem solver tended to accompany the roles of a protagonist and a teacher. This finding indicates that anticipating mathematical operations and structures is also important for making appropriate decisions in the roles of a protagonist and a teacher. Future research has to reveal which role the anticipation of mathematics that is important for the solution of the self-generated problem plays in modelling-related problem posing depending on the roles (preservice) teachers are adopting.

LIMITATIONS

Our study has some limitations that should be kept in mind when interpreting the results. The analysis was based on data from a small sample of preservice teachers who were prompted to pose mathematical problems based on given real-world situations. The roles identified in the problem posing processes seem to be specific to modelling-related problem posing with preservice teachers. Research has yet to determine which of the roles can be identified for different problem posing stimuli (e.g., problem posing based on given intramathematical situations) or other samples (e.g., school students). The transferability of the results needs to be validated in future studies.

CONCLUSION

Overall, our study contributes to research on modelling from a problem posing perspective. The results of our study have theoretical implications for research on modelling and problem posing by underlining the importance of roles and domains for decision making while posing problems based on the real-world situation. In order to pose problems, (preservice) teachers adopt different roles. It might be important to teach (preservice) teachers which roles may occur while posing problems that are based on given real-world situations. Further, stimulating them to reflect on the roles they are adopting can be fruitful for the development of high-quality problems. Therefore, the results of the study have to be kept in mind when teaching (preservice) teachers how to pose real-world problems.

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