SET THEORY COURSE WINTER TERM 2020-21, EXERCISE SHEET NO. 9, JAN 21, 2021

RALF SCHINDLER

Hand in by Jan 25, 2021.

In what follows, let M be a countable transitive model of ZFC, let $\mathbb{P} \in M$ be a partial order, and let g be $\mathbb{P}$-generic over M.

Problem 1. Suppose that $p \Vdash_M^{\mathbb{P}} \exists x\, \varphi(x, \tau_1, \ldots, \tau_k)$, where $p \in \mathbb{P}$, φ is a formula, and τ_1, ..., $\tau_k \in M^{\mathbb{P}}$. Show that there is some $\sigma \in M^{\mathbb{P}}$ such that $p \Vdash_M^{\mathbb{P}} \varphi(\sigma, \tau_1, \ldots, \tau_k)$. (This property is called "fullness.")

Problem 2. Let $\mathbb{P} = \{p \colon \exists \alpha < \aleph_\omega^M \; p \colon \alpha \to \aleph_{\omega+1}^M\}$, ordered by $p \leq q$ iff $p \supset q$. Show that $\aleph_{\omega+1}^M$ is countable in $M[g]$.

Problem 3. Let $S \subset \omega_1$ be stationary. Show that for every $\alpha < \omega_1$ there is some *closed* $c \subset S$ of order type $\alpha + 1$.

Problem 4. Let $S \subset \omega_1$ be stationary. Let $\mathbb{S} = \{p \colon \exists \alpha < \omega_1\, p \colon \alpha + 1 \to S \text{ is continuous }\}$. Show that $\mathbb{S}$ is σ-distributive.

INSTITUT FÜR MATHEMATISCHE LOGIK UND GRUNDLAGENFORSCHUNG, UNIVERSITÄT MÜNSTER, EINSTEINSTR. 62, 48149 MÜNSTER, GERMANY

URL: http://wwwmath.uni-muenster.de/logik/Personen/rds