Problem list. 1st Girona conference on inner model theory

1. (Schindler) Is there an extender model $L[E]$ whose mantle is not a fully iterable (from the point of view of $L[E]$) (hod) mouse?

2. (Goldberg) An irreducible ultrafilter is a countably complete ultrafilter whose ultrapower embedding doesn't factor as a finite linear iteration of ultrapowers by internal ultrafilters. Schlutzenberg-Steel: In $L[E]$, the irreducible ultrafilters are exactly the measures on the sequence.

Is it consistent with a κ^+ supercompact cardinal κ that the Mitchell order restricted to irreducible ultrafilters is a well-order? What about in fine structural models?

In $L[E]$, if there is a normal measure concentrating on α's which are α^+ supercompact, then there is a normal measure which is not on the sequence. (Woodin: "Fine strucure at the finite levels of supercompactness.")

3. Let (M, Σ) be a mouse pair, and let S be an iteration tree on M of successor length (not nec. by Σ) with a nice m-strategy Λ. Is $\Lambda = \Psi^*_S$ for some Ψ such that (M, Ψ) is a mouse pair?

Analoguous question for type 2.

4. (Aguilera) What is the consistency strength of the determinacy of the least σ-algebra containing all the projecive sets?

Lower bound: PD. Upper bound: For each $x \in \mathbb{R}$, $N^{\#}_{\omega+1}(x)$ exists. Here, $N^{\#}_{\omega+1}(x)$ is the least active sound mouse N with a Woodin cardinal δ such that for each $n < \omega$, $M^{\#}_n(N|\delta) \lhd N$.

5. (Wilson) What is the consistency strength of ZF plus "every Suslin set is (boldface) Σ^1_2"? (Add DC if you want.)

Upper bound: A generic Vopenka cardinal. Lower bound: ZFC.

6. (Schlutzenberg) Let $\kappa \geq \aleph_1$ be regular. Suppose M has a $\kappa + 1$ iteration strategy with strong hull condensation. $\mathbb{P}$ has the κ-c.c. implies that M is still $\kappa+1$ iterable in $V^{\mathbb{P}}$. (Schlutzenberg: "Iterability for stacks.") For which other forcings is this also true (or consistently false)?

Example: $\kappa = \omega_1$. Can have a proper forcing $\mathbb{P}$ such that $M^{\#}_1$ is not $\omega_1 + 1$ iterable in $V^{\mathbb{P}}$, while $M^{\#}_1$ is $\omega_1 + 1$ iterable in V. (Schindler-Schlutzenberg) What about a σ-closed $\mathbb{P}$?

7. (Mesken) Let Γ be a class of forcings. The Γ mantle is the intersection of all grounds W such that $W[g] = V$, g $\mathbb{P}$-generic over W for some $\mathbb{P} \in \Gamma^W$. For natural Γ, the Γ mantle is a model of ZF. (Fuchs-Hamkins-Reitz)

(1) Can the Γ mante be a model of AD? (Fuchs: Yes! Let $V = L(\mathbb{R})^{\mathrm{Col}(\omega_1,\mathbb{R})}$, $\Gamma = \sigma$-closed forcings.)

(2) What is the Γ mantle of a given $L[E]$ model? E.g., $L[E]$ being the least model with a strong above a Woodin, $\Gamma = \sigma$-closed posets.

(3) (Goldberg) Is there an $L[E]$ which has an inner model N such that $N[g] = L[E]$, where g is $\mathbb{P}$-generic over N for some non-trivial $\mathbb{P} \in N$ which is σ-closed (σ-distributive) in N?

8. (Steel) Let $M_1^\# \leq_T x$. Work in $L[x]$. Let N be M_1-like if N is an inner model, $N \models$ "I'm M_1," and $\delta^N < \omega_1$. Let H be the result of simultaneously pseudo comparing all such N. $H = L[\mathcal{M}(\mathcal{T})]$ for any tree $\mathcal{T}$ from that comparison. $\delta^H = \omega_2$.

Is $H|\omega_2 = HOD|\omega_2$?

9. (Adolf) Let M_{refl} be the least mouse with some λ, a limit of Woodin cardinals and $< \lambda$ strongs and some $\kappa < \lambda$ which is A-strong up to λ, where $A = \{\mu < \lambda \colon \mu$ is $< \lambda$ strong $\}$. Assume M_{refl} has a good $\omega_1 + 1$ iteration strategy.

(1) Does M_{refl} have a uniquely assigned derived model? E.g., does $D(M_{\mathrm{refl}}, < \lambda)$ satisfy lsa?

(2) (Steel) Let M_0, M_1 be $\mathbb{R}$-genericity iterates of M_{refl}. I.e., have g_0 and g_1 $\mathrm{Col}(\omega, < \omega_1^V)$-generic over M_0, M_1, respectively, (ω_1^V being the image of λ under the iteration maps) such that $\mathbb{R}^*_{g_0} = \mathbb{R}^V = \mathbb{R}^*_{g_1}$. Is $Hom^*_{g_0} = Hom^*_{g_1}$?

10. (Steel) Does AD^+ plus "no long extender" imply hod pair capturing?

11. (Goldberg) Assume $\mathrm{AD}^{L(\mathbb{R})}$. Does $L(\mathbb{R})$ satisfy the ground axiom?