A coarse Dodd-Jenett premouse is a model

\[ M = (L_\alpha(\mathcal{U}); \in, \mathcal{U}) \text{ s.t. } M \models ZFC^- (ZFC without the power set axiom) \text{ plus } M \models "\mathcal{U} \text{ is a normal } \kappa\text{-complete nonprincipal ultrafilter on } \kappa," \]

some \( \kappa \in M \).

If \( M \) is a coarse DJ premouse, we may define the ultrapower \( \text{ult}_M : M \rightarrow \text{ult}(M; \mathcal{U}) \) using functions in \( M \). \( \text{ult}(M; \mathcal{U}) \) will always be well-founded, so that we may identify it with a transitive structure; \( \text{ult}_M \) will be fully elementary, so that \( \text{ult}(M; \mathcal{U}) \) will again be a coarse DJ premouse.

**Lemma 1.** Let \( M = (L_\alpha(\mathcal{U}); \in, \mathcal{U}) \) be a coarse DJ premouse, let \( \mathcal{U} \) be on \( \kappa \), and let

\[ \text{ult}_M : M \rightarrow \text{ult}(M; \mathcal{U}) \]

be the ultrapower map.

Then \( \mathcal{P}(\kappa) \cap \text{ult}(M; \mathcal{U}) = \mathcal{P}(\mathcal{U}) \cap M \).
Proof: "\( \supset \)": If \( A \in \mathcal{P}(\kappa) \cap M \), then
\[
A = i_M^\kappa(A) \cap \kappa \in \text{ut}(M; \kappa).
\]
"\( \subset \)": Let \( A \in \mathcal{P}(\kappa) \cap \text{ut}(M; \kappa) \). Say \( A = i_M^\kappa(f)(\kappa) \), so that \( \exists \xi \in A \) iff \( \{ \gamma < \kappa : \xi \in f(\gamma) \} \in \kappa \). While \( A_\xi = \{ \gamma < \kappa : \xi \in f(\gamma) \} \in \kappa \).

By the proof of "\( \supset \)", \( (A_\xi : \xi < \kappa) \in \text{ut}(M; \kappa) \), in fact \( (A_\xi : \xi < \kappa) = (A_\xi' \cap \kappa : \xi < \kappa) \), where
\[
(A_\xi' : \xi < i_M^\kappa(\kappa)) = i_M^\kappa((A_\xi : \xi < \kappa)).
\]
Then \( \exists \xi \in A \) iff \( \{ \gamma < \kappa : \xi \in f(\gamma) \} \in \kappa \) iff \( \kappa \in i_M^\kappa(A_\xi) \) iff \( \kappa \in \text{the } \xi^{th} \text{ element of } i_M^\kappa((A_\xi : \xi < \kappa)) \), so \( A \in \text{ut}(M; \kappa) \). \( \Box \)

Given some coarse JD premouse, \( M, u \) may define a putative iteration
\[
(M_i, \tau_{ij} : i \leq j \leq \alpha)
\]
of length \( \alpha + 1 \) as usual. \( M \) is called

\underline{iterable} if \( M_\alpha \) is well-founded (transitive).
for all putative isradims \((M_i, \pi_{ij} : i \leq j \leq \alpha)\) of \(M\) of length \(\alpha\).

\(L[\kappa]\) itself is iterable, so that if \(\gamma > \text{crit}(U)\) is such that \(L[\gamma][U] \models \exists \kappa\) and if

\[\sigma : L[\kappa] \rightarrow L[\gamma][U]\]

is fully iterable, then \(L[\kappa] = (L[\kappa]; \epsilon, U)\) is an iterable coarse DJ premouse.

**Lemma 2. (Comparison)** Let \(M = (L[\kappa]; \epsilon, U)\) be an iterable coarse DJ premouse. Let \(\mu > \kappa\) be a regular cardinal (in \(V\)), and let \(\mathcal{F}_\mu\) denote the club filter on \(\mu\) (as being defined in \(V\)).

Let \((M_i, \pi_{ij} : i \leq j \leq \mu)\) be the isradim of \(M\) of length \(\mu\). There is then \(\alpha'\) s.t.

\[M' = (L[\kappa]; \epsilon, \mathcal{F}_\mu \cap L[\kappa]; \mathcal{F}_\mu).\]

Proof: Write \(\kappa_i = \text{crit}(U_i)\), where \(M_i = (L[\kappa_i]; \epsilon, U_i),\ i \leq \mu\).
If $i < \mu$, then $\pi_{i+1}(A) \in U_{i+1}$ iff $A \in U_i$, iff $\xi_i \in \pi_{i+1}(A)$, which implies that for all $i < j \leq \mu$, if $B \in \text{ran} (\pi_{ij})$, then $B \in U_j$ iff $\xi_i \in B$.

Then if $j \leq \mu$ is a limit ordinal, for all $B \in P(\xi_j) \cap M_j$,

$B \in U_j$ iff $\xi_i \in B$ for a tail end of $i < j$.

This implies $(\ast)$. 

**Corollary 3.** Let $M, M'$ be iterable coarse DJ premises. There is some $\mu$ s.t. if

$$(M_i, \pi_{ij} : i \leq j \leq \mu)$$

and the initial $M, M'$ resp. at level $\mu + 1$, then if $M_\mu = (L_{\alpha \cap U_{\mu}}, \epsilon, U_\mu)$, then $M' = $
\[(L, \{E_{\mu}\}; \varepsilon, u_{\mu}),\]

\[\alpha \leq \alpha' \text{ and } u_{\mu} = u_{\mu} \cap L_{\alpha}[E_{\mu}] \quad \text{or} \]
\[\alpha' \leq \alpha \text{ and } u_{\mu}' = u_{\mu} \cap L_{\alpha}[E_{\mu}'] .\]

Lemma 1 implies that if \( M = (L, \{E_{\mu}\}; \varepsilon, u) \) is an iterable coarse DJ premouse, if
\[(M_{i}, \pi_{ij}; i \leq j < \mu),\]
is an iteration of \( M \) of length \( \mu \), and if \( \gamma < \text{crit}(u) \), then
\[\varphi(\gamma) \cap M_{\mu} = \varphi(\gamma) \cap M .\]

Corollary 3 then yields:

Corollary 4. Let \( M, M' \) be iterable coarse
DJ premice, ad let \( \gamma < \min(\text{crit}(u), \text{crit}(u')) \),
when \( U, U' \) is the measure of \( M, M' \), resp.;
then \( \varphi(\gamma) \cap M \subseteq M' \) or \( \varphi(\gamma) \cap M' \subseteq M \).

For any coarse DJ premouse \( M \), we may let \( \leq_{M} \).
denote the order of constructibility of \( M \).

If \( A \in M \), we let \( \text{pred}_{<M} (A) \) denote \( \{ x \in M : x <_M A \} \).

Lemma 5. Let \( M = (L_{\aleph_1}, \in, \mathcal{U}) \) be an iterable coarse DJ premouse, and let \( \gamma < \alpha \). Let \( A \in \mathcal{P}(\gamma) \cap M \). Then

\[
\text{Card} \left( \text{pred}_{<M} (A) \cap \mathcal{P}(\gamma) \right) \leq \gamma
\]

inside any transitive 2FC-model \( P \) with \( M \in P \).

Proof: Let

\[
\sigma : \bar{M} \rightarrow M
\]

be fully elementary and that \( (\gamma + 1) \cup \{ A \} \subseteq \text{ran}(\sigma) \), \( \bar{M} \) transitive, and \( \text{Card}(\bar{M}) = \gamma \).

By Cor. 4,

\[
\text{pred}_{<M} (A) \cap \mathcal{P}(\gamma) \subseteq \bar{M}.
\]

Corollary 6. \( L[\mathcal{U}] \models \text{GCH} \).
Lemma 8. \( L(\mathfrak{u}) \models \text{there is a } \Delta^1_3 \text{ well-ordering of } \mathfrak{u} \).

Proof: For \( x, y \in \mathfrak{u} \), \( x \lesssim_{L(\mathfrak{u})} y \) iff there is a (stable) iterable \( \mathcal{J} \) premouse \( M \) with \( x \lesssim_M y \). Being a stable iterable \( \mathcal{J} \) premouse is \( \text{IT}^1_2 \) in the codes. \( \square \)

Lemma 8. \( L(\mathfrak{u}) \models \left( \forall \mu \right) \Diamond \mu \) for all regular \( \mu \).

Proof: We work inside \( L(\mathfrak{u}) = M \).

Let us define \( (C_\xi, A_\xi : \xi < \mu) \) recursively as follows.

\( (C_\xi, A_\xi) = \) the \( \preceq_M \)-least \( (C, A) \) s.t. \( C \preceq \xi \) is club, \( A \preceq \xi \), and
\[
\{ \overline{\xi} < \xi : A \cap \overline{\xi} = A_{\overline{\xi}} \} \cap C = \emptyset.
\]

We claim that \( (A_\xi : \xi < \mu) \) witnesses \( \Diamond \mu \).

Otherwise let \( (C, A) \) be the \( \preceq_M \)-least counterexample.
Let \( \sigma : \bar{M} \to H^M_\theta \) be fully elementary, 
when \( \theta \) is regular and large enough, \( \text{Card}(\bar{M}) \leq \mu \), 
\( \text{ran}(\sigma) \cap \mu \in \mu \), \( \{ \mu, C, A \} \subseteq \text{ran}(\sigma) \). Set \( \bar{\mu} = \sigma^{-1}(\mu) \).

Then \( (C \cap \bar{\mu}, A \cap \bar{\mu}) = (C, A) \), so \( \bar{\mu} \in \{ \bar{\xi} < \mu : A \cap \bar{\xi} = A_{\bar{\xi}} \} \cap C \), and then \( \{ \bar{\xi} < \mu : A \cap \bar{\xi} = A_{\bar{\xi}} \} \cap C_\mu \neq \emptyset \).

Lemma 9. \( L(EU) \models \langle \Box^*_\mu \rangle \) for all infinite \( \mu \).

Proof: We will assume \( L(EU) = M \).

For \( \bar{\xi} < \mu^+ \), let \( A_{\bar{\xi}} = \bigcup \{ A(\bar{\xi}) \cap \bar{M} : \bar{M} \text{ is an iterable coarse } DJ \text{ premouse with} \} \).

It can be shown that in \( L(EU) \), \( \Box^{\mu}_\mu \) holds if \( \mu \) is not ineffable.