Errata of the book
"Set theory. Exploring Independence and Truth"

by Ralf Schindler

p.2 l.-6: delete the first "u" in "analoguous."

p.4, l.8f.: the definition should read: "A set $B \subset A$ is called *dense in A* iff for all a, $b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read "Proof of Theorem 1.9," not "Proof of Theorem 1.8."

p.5 l.16: delete the last "that."

paragraph at the bootom of p.6 and the top of p.7: delete the sentence "As $\mathbb{Q}$ is dense [...] picked to be pairwise disjoint."

p.8 l.-2: delete "$[a, b]_\infty$ is dense in $[a, b]$." This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that b does *not* have a maximum [...].

p.18 l.-7: delete "the."

p.20 l.10: Shat *that* [...]

p.23 l.8: insert "is" before "inductive."

p.27 l.9: replace "the R–least x_0" by "an R–least x_0." (Thanks to Philipp Schlicht!)

p.34 l.1: "my" should be "may."

p.35 l.3f.: ... for cardinals κ, λ *with* $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(\aleph_0)$ by $\pi((\aleph_0, \aleph_0))$, l.27: relace $\pi(\aleph_\alpha)$ by $\pi((\aleph_\alpha, \aleph_\alpha))$.

p.37 l.16: replace "is" by "in."

p.38 l.7: replace "Poblem" by "Problem."

p.41 l.-3: replace $A_{x\eta_n}$ by A_{η_n}.

p.43 l.1: replace "from" by "form."

p.43 l.2: replace κ^+ by κ.

p.44 l.3: replace $\gamma_\alpha^{\alpha'} \bar{S}$ by $\gamma_\alpha^{\alpha'} \in \bar{S}$.

p.44 l.-10: replace $g_i \colon [\mu_i]^{\mathrm{cf}(\kappa)} \to \mu_i^+$ by $g_i \colon [\mu_i]^{\leq \mathrm{cf}(\kappa)} \to \mu_i^+$.

p.44 l.-3: unfortunately, this is not the same g_i as in l.-10 of the same page.

p.45 l.2: delete "in."

p.45 l.11 and l.15: replace $\mathcal{P}(\kappa)$ by $[\kappa]^{\mathrm{cf}(\kappa)}$; also lines 11, 14, 15, and 16: replace $Y \subset \kappa$ by $Y \in [\kappa]^{\mathrm{cf}(\kappa)}$.

p.49f.: The construction of a κ^+ Aronszajn tree is imprecise. Let us fix it as follows.

Let $(A_s \colon s \in {}^{<\omega}\kappa)$ be such that $A_\emptyset = \kappa$ and for all $s \in {}^{<\omega}\kappa$, $\{A_{s^\frown\xi} \colon \xi < \kappa\}$ is a family of pairwise disjoint sets with $A_s = \bigcup\{A_{s^\frown\xi} \colon \xi < \kappa\}$. Let

$$\mathcal{A} = \{\cup B \colon B \in [\{A_s \colon s \in {}^{<\omega}\kappa \wedge s \neq \emptyset\}]^{<\kappa}\}.$$

By $\kappa^{<\kappa} = \kappa$, $\mathrm{Card}(\mathcal{A}) = \kappa$.

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all $s \in T$ there is some $A \in \mathcal{A}$ with $\mathrm{ran}(s) \subset A$.

(2) If $s \in T$, $\mathrm{ran}(s) \subset A \in \mathcal{A}$, $B \in \mathcal{A}$, $B \cap A = \emptyset$, $\mathrm{lv}_T(s) < \beta < \kappa^+$, then there is some $t \in T$ with $\mathrm{lv}_T(t) = \beta$, $s \subset t$, and $\mathrm{ran}(t) \subset \mathrm{ran}(s) \cup B$.

(4) Let $\lambda < \kappa^+$ be a limit ordinal with $\mathrm{cf}(\lambda) < \kappa$. Let $C \subset \lambda$ be club in λ with $\mathrm{otp}(C) = \mathrm{cf}(\lambda)$, and let $(\lambda_i \colon i < \mathrm{cf}(\lambda))$ be the monotone enumeration of $\{0\} \cup C$. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be a pairwise disjoint family of elements of $\mathcal{A}$. Let $s \colon \lambda \to \kappa$ be such that

$$s \restriction \lambda_i \in T_{\lambda_i+1} \wedge s``\lambda_i \subset \bigcup\{A_j \colon j < i\}$$

for every $i < \mathrm{cf}(\lambda)$. Then $s \in T_{\lambda+1}$.

The rest is as before except that in case $\mathrm{cf}(\lambda) = \kappa$ we pick $s(t)$ as follows. We fix $C \subset \lambda$, a club in λ with $\mathrm{otp}(C) = \kappa$, and we let $(\lambda_i \colon i < \kappa)$ be the monotone enumeration of $\{0\} \cup C$. By the new (1), $\mathrm{ran}(s) \subset A \in \mathcal{A}$ for some A. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be such that $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{A, B\}$ is a pairwise disjoint family of elements of $\mathcal{A}$. (This choice is possible!) Using the new (2) and the new (4), we may construct some $t \colon \lambda \to \kappa$ extending s such that for every $i < \kappa$,

$$t \restriction \lambda_i \in T_{\lambda_i+1} \wedge t``\lambda_i \subset A \cup \bigcup\{A_j \colon j < i\}.$$

We write $t(s)$ for this t. We then let $T_{\lambda+1} = T_\lambda \cup \{t(s) \colon s \in T_\lambda\}$.

p.62, Problem 4.4: cf. p.35 l.3f.

p.97 footnote 1: replace "until p. 97" by "until p. 101."

p.106 l.19: add ", and $\{\xi^p_{k_0} \colon p \in D_1\}$ is unbounded in ω_1." (Let $\beta < \omega_1$ be such that $\xi^p_k < \beta$ for all $p \in D_0$ and $1 \leq k < k_0$. If for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and for all $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$ the set $\{\xi^p_{k_0} \colon p \in D_0 \wedge \xi^p_1 = \xi_1 \wedge \ldots \wedge \xi^p_{k_0-1} = \xi_{k_0-1} \wedge p(\xi_1) = s_1 \wedge \ldots \wedge p(\xi_{k_0-1}) = s_{k_0-1}\}$ were bounded in ω_1, then there would be one common bound for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$, contradicting the choice of k_0.) In l.23f., replace "By the choice of k_0" with "By the choice of D_1."

p.139 l.7 from b.: This should say "Also, if $(2^{\aleph_0})^{L[x]} = \omega_1^{L[x]} < 2^{\aleph_0}$, then by Lemmas 7.19 and 7.20 there is a *largest* $\Sigma^1_2(x)$–set of reals which is smaller than $2^{\aleph_0}$, namely ${}^\omega\omega \cap L[x]$."

p.140: the 2nd last displayed formula on that page got screwd up. The aim is to choose x such that $(\varphi_0(x), x(0), \varphi_1(x), x(1), \ldots)$ is $<_{\mathrm{lex}}$–minimal. Let the formula read:

$$[x \restriction n = y \restriction n \wedge \forall m < n(\varphi_m(x) = \varphi_m(y))] \longrightarrow$$

$$[y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y) \vee (\varphi_n(x) = \varphi(y) \wedge x(n) \leq y(n)))].$$

As explained in the text, for $x \in A$, "$y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y)))$" and "$y \notin A \vee (y \in A \wedge (\varphi_n(x) \leq \varphi_n(y)))$" can both be uniformly written in a Π^1_1 as well as in a Σ^1_1 way, so that the relevant formula is Π^1_1. (Thanks to Robin Puchalla!)

p.210 Definition 10.45: It has to be added that if E is a (κ, ν)–extender, then ν is called the *length of* E. The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)

p.226: U^* refers to two different things on this page, to a tree, defined l.9, and to a substructure of R_i, defined l.17 (display). Also, τ refers to two different things on this page, to $\sigma' \restriction V^{\bar{M}_i}_{\nu_i}$, defined l.9, and to a map from (the 2nd) U^* to $V^{R_i}_\kappa$, defined l.18. There is also a sloppyness about Σ_{1+} formulae on this page in that the first parameter (free variable) of Φ got suppressed: e.g. in (10.46) by $\Phi(\sigma_i \restriction V^{\bar{M}_i}_{\nu_i})$ I really meant $\Phi(\sigma_i(\nu_i), \sigma_i \restriction V^{\bar{M}_i}_{\nu_i})$, i.e., $\Phi(\tau)$ in l.7 should have been written as $\Phi(\tau(\nu_i), \tau)$ – with the understanding that $\tau(\nu_i) = \sup(\tau"\nu_i)$. (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make "$\forall x \in U' \, \exists y \in U' \, x \in y$" part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take $U = 4$, $U' = 4 \cup \{\{0, 2\}\}$, and $\pi = \mathrm{id}$. (Thanks to Toby Meadows!)

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.