Errata of the book
“Set theory. Exploring Independence and Truth”
by Ralf Schindler

p.2 l.-6: delete the first “u” in “analogous.”

p.4, l.8f.: the definition should read: “A set $B \subset A$ is called dense in $A$ iff for all $a, b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read “Proof of Theorem 1.9,” not “Proof of Theorem 1.8.”

p.5 l.16: delete the last “that.”

paragraph at the bottom of p.6 and the top of p.7: delete the sentence “As $\mathbb{Q}$ is dense [...] picked to be pairwise disjoint.”

p.8 l.-2: delete “[a, b]_{\infty}$ is dense in [a, b].” This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that $b$ does not have a maximum [...].

p.18 l.-7: delete “the.”

p.20 l.10: Shat that [...]

p.23 l.8: insert “is” before “inductive.”

p.27 l.9: replace “the $R$–least $x_0$” by “an $R$–least $x_0$.” (Thanks to Philipp Schlicht!)

p.34 l.1: “my” should be “may.”

p.35 l.3f.: ... for cardinals $\kappa, \lambda$ with $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(\aleph_0)$ by $\pi((\aleph_0, \aleph_0)), 1.27: replace \pi(\aleph_0) by \pi((\aleph_0, \aleph_0))$.

p.37 l.16: replace “is” by “in.”

p.38 l.7: replace “Poblem” by “Problem.”

p.41 l.-3: replace $A_{\kappa\eta}$ by $A_{\eta \kappa}$.

p.43 l.1: replace “from” by ”form.”

p.43 l.2: replace $\kappa^+$ by $\kappa$.

p.44 l.3: replace $\gamma_{\alpha} \in \mathcal{S}$ by $\gamma_{\alpha}^{\mathcal{S}} \in \mathcal{S}$.

p.44 l.10: replace $g_i : [\mu_i]^{cf(\kappa)} \rightarrow \mu_i^+$ by $g_i : [\mu_i]^{\leq cf(\kappa)} \rightarrow \mu_i^+$.

p.44 l.-3: unfortunately, this is not the same $g_i$ as in l.-10 of the same page.

p.45 l.2: delete “in.”
p.45 l.11 and l.15: replace $\mathcal{P}(\kappa)$ by $[\kappa]^{\text{cf}(\kappa)}$; also lines 11, 14, 15, and 16: replace $Y \subset \kappa$ by $Y \in [\kappa]^{\text{cf}(\kappa)}$.

p.49f.: The construction of a $\kappa^+$ Aronszajn tree is imprecise. Let us fix it as follows.

Let $(A_s: s \in <\omega \kappa)$ be such that $A_\emptyset = \kappa$ and for all $s \in <\omega \kappa$, $\{A_s \setminus \xi: \xi < \kappa\}$ is a family of pairwise disjoint sets with $\text{otp}(s)$. Let

$$A = \{\cup B: B \in \{\{A_s: s \in <\omega \kappa \land s \neq \emptyset\}\}^{<\kappa}\}.$$ 

By $\kappa < \kappa = \text{Card}(A) = \kappa$.

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all $s \in T$ there is some $A \in \mathcal{A}$ with $\text{ran}(s) \subset A$.

(2) If $s \in T$, ran$(s) \subset A \in \mathcal{A}$, $B \in \mathcal{A}$, $B \cap A = \emptyset$, lv$_T(t) < \beta < \kappa^+$, then there is some $t \in T$ with lv$_T(t) = \beta$, $s \subset t$, and $\text{ran}(t) \subset \text{ran}(s) \cup B$.

(4) Let $\lambda < \kappa^+$ be a limit ordinal with $\text{cf}(\lambda) < \kappa$. Let $C \subset \lambda$ be club in $\lambda$ with $\text{otp}(C) = \text{cf}(\lambda)$, and let $(\lambda_i: i < \text{cf}(\lambda))$ be the monotone enumeration of $\{0\} \cup C$. Let $\{A_i: i < \text{cf}(\lambda)\} \cup \{B\}$ be a pairwise disjoint family of elements of $\mathcal{A}$. Let $s: \lambda \to \kappa$ be such that

$$s \upharpoonright \lambda_i \in T_{\lambda_i+1} \land s^*\lambda_i \subset \bigcup \{A_j: j < i\}$$

for every $i < \text{cf}(\lambda)$. Then $s \in T_{\lambda+1}$.

The rest is as before except that in case $\text{cf}(\lambda) = \kappa$ we pick $s(t)$ as follows. We fix $C \subset \lambda$, a club in $\lambda$ with $\text{otp}(C) = \kappa$, and we let $(\lambda_i: i < \kappa)$ be the monotone enumeration of $\{0\} \cup C$. By the new (1), $\text{ran}(s) \subset A \in \mathcal{A}$ for some $A$. Let $\{A_i: i < \text{cf}(\lambda)\} \cup \{B\}$ be such that $\{A_i: i < \text{cf}(\lambda)\} \cup \{A, B\}$ is a pairwise disjoint family of elements of $\mathcal{A}$. (This choice is possible!) Using the new (2) and the new (4), we may construct some $t: \lambda \to \kappa$ extending $s$ such that for every $i < \kappa$, $t \upharpoonright \lambda_i \in T_{\lambda_i+1} \land t^*\lambda_i \subset A \cup \bigcup \{A_j: j < i\}$.

We write $t(s)$ for this $t$. We then let $T_{\lambda+1} = T_\lambda \cup \{t(s): s \in T_\lambda\}$.

p.62, Problem 4.4: cf. p.35 l.3f.

p.97 footnote 1: replace “until p. 97” by “until p. 101.”

p.106 l.19: add “, and $\{\xi^p_{k_0}: p \in D_1\}$ is unbounded in $\omega_1$.” (Let $\beta < \omega_1$ be such that $\xi^p_{k_0} < \beta$ for all $p \in D_0$ and $1 \leq k < \kappa_0$. If for all $\xi_1 < \ldots < \xi_{k_0-1} < \beta$ and for all $s_1, \ldots, s_{k_0-1} \in <\omega_\omega$ the set $\{\xi^p_{k_0}: p \in D_0 \land \xi^p_{k_0} = \xi_1 \land \ldots \land \xi^p_{k_0-1} = \xi_{k_0-1} \land p(\xi_1) = s_1 \land \ldots \land p(\xi_{k_0-1}) = s_{k_0-1}\}$ were bounded in $\omega_1$, then there would be one common bound for all $\xi_1 < \ldots < \xi_{k_0-1} < \beta$ and $s_1, \ldots, s_{k_0-1} \in <\omega_\omega$, contradicting the choice of $\kappa_0$.) In l.23f., replace “By the choice of $\kappa_0$” with “By the choice of $D_1$.”

p.137f., proof of Claim 7.18: The construction of $U$ is wrong. Let $\delta, \epsilon \mapsto (\delta, \epsilon)$ be the GÖDEL pairing function, see p.35. Then let $(s, t) \in U$ iff for all $n < \omega$, $(s \upharpoonright k, n, (t((n(n, 0), \ldots, t((n, k-1))) < \omega, k \geq 0$ is maximal such that $k = 0$ or $(n, k-1) < \text{lh}(t) = \text{lh}(s)$. (Thanks to Andreas Lietz and Stefan Hoffelner!)
p.139 l.7 from b.: This should say “Also, if \( (2^{2^{80}})^{L[x]} = \omega_1^{L[x]} < 2^{80} \), then by Lemmas 7.19 and 7.20 there is a largest \( \Sigma^1_2(x) \)-set of reals which is smaller than \( 2^{80} \), namely \( \omega \cap L[x] \).”

p.140: the 2nd last displayed formula on that page got screwed up. The aim is to choose \( x \) such that \( \langle \varphi_0(x), x(0), \varphi_1(x), x(1), \ldots \rangle \) is \( \varphi \text{-} \text{lex-minimal} \). Let the formula read:

\[
[x \mid n = y \mid n \wedge \forall m < n(\varphi_m(x) = \varphi_m(y))] \rightarrow
\]

\[
[y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y) \vee (\varphi_n(x) = \varphi(y) \wedge x(n) \leq y(n))))].
\]

As explained in the text, for \( x \in A \), “\( y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y))) \)” and “\( y \notin A \vee (y \in A \wedge (\varphi_n(x) \leq \varphi_n(y))) \)” can both be uniformly written in a \( \Pi^1_1 \) way, so that the relevant formula is \( \Pi^1_1 \). (Thanks to Robin Puchalla!)

p.162 last two lines: The proof of \( N \models \text{DC} \) has to be fixed as follows. Let \( R, a \in N \) be such that for all \( x \in a \) there is a \( y \in a \) with \( (x, y) \in R \). Let us work in \( V[G] \) and define \( f: \omega \rightarrow a \) as follows. For each \( n < \omega \),

\[
f(n) = \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi_n(u, \beta_n, x_n) \},
\]

where \( (\varphi_n, \alpha_n, \beta_n) \) is the \( \text{lex}-\text{least} \) \( (\varphi, \alpha, \beta) \) such that there is some \( y \in \omega \) such that

(a) \( \{ u \in V_\alpha : V_\alpha \models \varphi(u, \beta, y) \} \in a \), and

(b) if \( n > 0 \), then \( (x_{n-1}, \{ u \in V_\alpha : V_\alpha \models \varphi(u, \beta, y) \}) \in R \)

and \( x_n \in \omega \) is such that

(a) \( \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n) \} \in a \), and

(b) if \( n > 0 \), then \( (x_{n-1}, \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n) \}) \in R \).

Let \( x = \oplus_{n<\omega} x_n \). It is then easy to see that \( f \) is definable in \( V[G] \) from \( x \) together with an ordinal and a real parameter which defines \( (a, R) \). Hence \( f \in N \).

p.170 l.14: replace \( C \cap U_s \cap O^*_n = \emptyset \) by \( C \cap U_s \cap O^*_n \neq \emptyset \). (Thanks to Vincenzo Dimonte!)

p.180 l.2 from b.: replace \( g(n) \leq m < g(n+1) \) by \( f(n) \leq m < f(n+1) \)

p.195 l.6 from b.: replace \( \{ \xi < \kappa : f(x) \in y \} \) by \( \{ \xi < \kappa : f(\xi) \in y \} \). (Thanks to Vincenzo Dimonte!)

p.199 l.7 from b.: replace “Lemma 1.31 (g) and (e)” by “Lemma 10.21 (g) and (e).” (Thanks to Vincenzo Dimonte!)

p.199 l.2 from b.: replace “Lemma 10.29 (h)” by “Lemma 10.21 (h).” (Thanks to Vincenzo Dimonte!)

p.203 l.3 from b.: replace “In (e)” by “In (iii)” (Thanks to Vincenzo Dimonte!)

p.210 Definition 10.45: It has to be added that if \( E \) is a \( (\kappa, \nu) \)-extender, then \( \nu \) is called the \textit{length} of \( E \). The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)
p.226: $U^*$ refers to two different things on this page, to a tree, defined l.9, and to a substructure of $R_i$, defined l.17 (display). Also, $\tau$ refers to two different things on this page, to $\sigma'^{|V_{\bar{\nu}^i}}$, defined l.9, and to a map from (the 2nd) $U^*$ to $V_{\bar{\kappa}^i}$, defined l.18. There is also a sloppyness about $\Sigma_{1+}$ formulae on this page in that the first parameter (free variable) of $\Phi$ got suppressed: e.g. in (10.46) by $\Phi(\sigma_i | V_{\bar{\nu}^i})$ I really meant $\Phi(\sigma_i(\nu_i), \sigma_i | V_{\bar{\nu}^i})$, i.e., $\Phi(\tau)$ in l.7 should have been written as $\Phi(\tau(\nu_i), \tau)$ -- with the understanding that $\tau(\nu_i) = \sup(\tau'' \nu_i)$. (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make $\forall x \in U' \exists y \in U' x \in y$ part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take $U = 4$, $U' = 4 \cup \{\{0, 2\}\}$, and $\pi = \text{id}$. (Thanks to Toby Meadows!)

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.

p.306 l.-12: replace $\{\pi_{s,x}(\kappa_n): n < \omega, s \subset x, \text{lh}(s) = n + 1\}$ by $\{\pi_{s,x}(\kappa_n): n < \omega, s \subset x, \text{lh}(s) \geq n + 1\}$. (Thanks to Andreas Lietz!)