Errata of the book
“Set theory. Exploring Independence and Truth”
by Ralf Schindler

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p.2 l.-6: delete the first “u” in “analogous.”

p.4, l.8f.: the definition should read: “A set $B \subset A$ is called dense in $A$ iff for all $a, b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read “Proof of Theorem 1.9,” not “Proof of Theorem 1.8.”

p.5 l.16: delete the last “that.”

paragraph at the bottom of p.6 and the top of p.7: delete the sentence “As $\mathbb{Q}$ is dense [...] picked to be pairwise disjoint.”

p.8 l.2: delete “$[a, b]_{\infty}$ is dense in $[a, b]$.” This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that $b$ does not have a maximum [...].

p.18 l.7: delete “the.”

p.20 l.10: Shat that [...]”

p.23 l.8: insert “is” before “inductive.”

p.27 l.9: replace “the $R$–least $x_0$” by “an $R$–least $x_0$.” (Thanks to Philipp Schlicht!)

p.34 l.1: “my” should be “may.”

p.35 l.3f.: ... for cardinals $\kappa, \lambda$ with $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(\mathbb{N}_0)$ by $\pi((\mathbb{N}_0, \mathbb{N}_0))$, l.27: replace $\pi(\mathbb{N}_\alpha)$ by $\pi((\mathbb{N}_\alpha, \mathbb{N}_\alpha))$.

p.37 l.16: replace “is” by “in.”

p.38 l.7: replace “Poblem” by “Problem.”

p.41 l.-3: replace $A_{x\eta}$ by $A_{\eta}$.

p.43 l.1: replace “from” by “form.”

p.43 l.2: replace $\kappa^+$ by $\kappa$.

p.44 l.3: replace $\gamma_\alpha^S \bar{S}$ by $\gamma_\alpha^S \in \bar{S}$.

p.44 l.10: replace $g_i: [\mu_i]^{\aleph(\kappa)} \to \mu_i^+$ by $g_i: [\mu_i]^{\leq \aleph(\kappa)} \to \mu_i^+$.  

1
By \( \kappa \leq \kappa = \kappa \), \( \text{Card}(A) = \kappa \).

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all \( s \in T \) there is some \( A \in A \) with \( \text{ran}(s) \subseteq A \).

(2) If \( s \in T \), \( \text{ran}(s) \subseteq A \in A \), \( B \in A \), \( B \cap A = \emptyset \), \( \text{lv}_T(s) < \beta < \kappa^+ \), then there is some \( t \in T \) with \( \text{lv}_T(t) = \beta \), \( s \subseteq t \), and \( \text{ran}(t) \subseteq \text{ran}(s) \cup B \).

(3) Let \( \lambda < \kappa^+ \) be a limit ordinal with \( \text{cf}(\lambda) < \kappa \). Let \( C \subseteq \lambda \) be club in \( \lambda \) with \( \text{otp}(C) = \text{cf}(\lambda) \), and let \( \{ \lambda_i : i < \text{cf}(\lambda) \} \) be the monotone enumeration of \( \{0\} \cup C \). Let \( \{ A_i : i < \text{cf}(\lambda) \} \) \( \cup \) \{ \( B \) \} be a pairwise disjoint family of elements of \( A \). Let \( s : \lambda \rightarrow \kappa \) be such that

\[
s \upharpoonright \lambda_i \in T_{\lambda + 1} \land s^\kappa \lambda_i \subseteq \bigcup \{ A_j : j < i \}
\]

for every \( i < \text{cf}(\lambda) \). Then \( s \in T_{\lambda + 1} \).

The rest is as before except that in case \( \text{cf}(\lambda) = \kappa \) we pick \( s(t) \) as follows. We fix \( C \subseteq \lambda \), a club in \( \lambda \) with \( \text{otp}(C) = \kappa \), and we let \( \{ \lambda_i : i < \kappa \} \) be the monotone enumeration of \( \{0\} \cup C \). By the new (1), \( \text{ran}(s) \subseteq A \in A \) for some \( A \). Let \( \{ A_i : i < \text{cf}(\lambda) \} \) \( \cup \) \{ \( B \) \} be such that \( \{ A_i : i < \text{cf}(\lambda) \} \) \( \cup \) \( \{ A \} \) is a pairwise disjoint family of elements of \( A \). (This choice is possible!) Using the new (2) and the new (4), we may construct some \( t : \lambda \rightarrow \kappa \) extending \( s \) such that for every \( i < \kappa \),

\[
t \upharpoonright \lambda_i \in T_{\lambda + 1} \land t^\kappa \lambda_i \subseteq A \cup \bigcup \{ A_j : j < i \}.
\]

We write \( t(s) \) for this \( t \). We then let \( T_{\lambda + 1} = T_{\lambda} \cup \{ t(s) : s \in T_{\lambda} \} \).

p.54 l.8: \( \{ \xi < \kappa : f(\xi) \in g(\xi) \} \subseteq U \)

p.56 l.4: Theorem
p.62, Problem 4.4: cf. p.35 l.3f.

p.64, Problem 4.25: This is wrong for trivial reasons. The statement has to be adjusted as follows. If \( \kappa \) is ineffable, then there is no slim \( \kappa \)-KUREPA tree. Here, a \( \kappa \)-tree \( T \) is slim iff for all \( \alpha < \kappa \), \( T_\alpha = \{ s \in T : \text{lv}_T(s) < \alpha \} \) has size at most \( \text{Card}(\alpha) \). (Thanks to Shervin Sorouri!)

p.97 footnote 1: replace “until p. 97” by “until p. 101.”

pp.72f. and p.88: The proof of Lemma 5.11 is wrong, as not every rud\( E \) function is simple in the sense of the definition of “simple” given in “\( \subset \)" of that proof. E.g.
if \( f(\vec{x}) \) is \( \text{rud}_E \), then the formula \( f(\vec{x}) \in E \) need not be \( \Sigma_0 \) in the language \( \mathcal{L}_{E,E} \).

(Thanks to Shervin Sorouri!)

This proof should be fixed as follows. First,

\[
\mathcal{P}(U) \cap \Sigma_0^\omega (U; \varepsilon, E) = \mathcal{P}(U) \cap \Sigma_0^\omega (U \cup \{U, E\}; \varepsilon),
\]

so that we have to prove that

\[
\mathcal{P}(U) \cap \text{rud}_E(U \cup \{U\}) = \mathcal{P}(U) \cap \Sigma_0^\omega (U \cup \{U, E\}; \varepsilon).
\]

In the proof of “\( \subset \)”, the definition of “simple” then has to be adjusted as follows:

Let a formula \( \varphi \) in \( \mathcal{L}_{E,E} \) be \( \Sigma_0^\prime \) iff \( \varphi \) is in the smallest class \( \Gamma \) of formulas such that

(a) all atomic formulas are in \( \Gamma \),

(b) \( \Gamma \) is closed under sentential connectives,

(c) \( \Gamma \) is closed under bounded quantification, and

(d) if \( \psi \) is in \( \Gamma \) and \( x \) is a variable, then \( \exists x \in E \psi \) and \( \forall x \in E \psi \) are both in \( \Gamma \).

These are the closure conditions as in Definition 5.1 on p.67 plus that quantification over elements of \( E \) counts as bounded quantification.

Now call a function \( f : V^k \to V \), where \( k < \omega \), simple iff the following holds true:

- \( \varphi(v_0, v_1, \ldots, v_m) \) is \( \Sigma_0^\prime \) in the language \( \mathcal{L}_{E,E} \), then \( \varphi(f(v'_1, \ldots, v'_m), v_1, \ldots, v_m) \) is equivalent over \( \text{rud}_E \) closed structures to a \( \Sigma_0^\prime \) formula in the same language.

This is also the definition of “simple” which should be used in Problem 5.8 on p.88.

p.106 l.19: add “, and \( \{\xi_k^p : p \in D_1\} \) is unbounded in \( \omega_1 \)” (Let \( \beta < \omega_1 \) be such that \( \xi_k^p < \beta \) for all \( p \in D_0 \) and \( 1 \leq k < k_0 \). If for all \( \xi_1 < \ldots < \xi_{k_0-1} < \beta \) and for all \( s_1, \ldots, s_{k_0-1} \in \omega \), the set \( \{\xi_k^p : p \in D_0 \land \xi_1 = \xi_1 \land \ldots \land \xi_{k_0-1} = \xi_{k_0-1} \land p(\xi_1) = s_1 \land \ldots \land p(\xi_{k_0-1}) = s_{k_0-1}\} \) were bounded in \( \omega_1 \), then there would be one common bound for all \( k_1 < \ldots < k_{k_0-1} < \beta \) and \( s_1, \ldots, s_{k_0-1} \in \omega \), contradicting the choice of \( k_0 \).) In l.23f., replace “By the choice of \( k_0 \)” with “By the choice of \( D_1 \).”

p.118 Corollary 6.62: Add “\( \mathcal{P} \) is separative” to the hypotheses in the statement of this corollary, as its proof makes use of Lemma 6.60. (Thanks to Fan Feng!)

p.120 l.5 from b.: replace “Lemma 6.65” by “Lemma 6.32.” (Thanks to Fan Feng!)

p.137f., proof of Claim 7.18: The construction of \( U \) is wrong. Let \( \delta, \epsilon \mapsto (\delta, \epsilon) \) be the Gödel pairing function, see p.35. Then let \( (s, t) \in U \) iff for all \( n < \omega \), \( (s \upharpoonright k, n \upharpoonright ((n, n), \ldots, t((n, k-l)))) \in U \), where \( k \geq 0 \) is maximal such that \( k = 0 \) or \( (n, k-l) < \text{lh}(t) = \text{lh}(s) \). (Thanks to Andreas Lietz and Stefan Hoffelner!)

p.139 l.7 from b.: This should say “Also, if \( (2^\aleph_0)^{L[x]} = \omega_1^{L[x]} < 2^\aleph_0 \), then by Lemmas 7.19 and 7.20 there is a largest \( \Sigma_2^1(x) \)-set of reals which is smaller than \( 2^\aleph_0 \), namely \( \omega \cap L[x] \).”
As explained in the text, for \( x \) and \( n \) such that 
\[
y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y) \lor (\varphi_n(x) = \varphi(y) \land x(n) \leq y(n))).
\]
As explained in the text, for \( x \in A \), \( y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y))) \) and “\( y \notin A \lor (y \in A \land (\varphi_n(x) \leq \varphi_n(y))) \)” can both be uniformly written in a \( \Pi^1_1 \) way as well as in a \( \Sigma^1_1 \) way, so that the relevant formula is \( \Pi^1_1 \). (Thanks to Robin Puchalla!)

p.162 last two lines: The proof of \( N \models \text{DC} \) has to be fixed as follows. Let \( R \), \( a \in N \) be such that for all \( x \in a \) there is a \( y \in a \) with \( (x,y) \in R \). Let us work in \( V[G] \) and define \( f : \omega \to a \) as follows. For each \( n < \omega \),
\[
f(n) = \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi_n(u, \beta_n, x_n) \},
\]
where \( (\forall \varphi \gamma, \alpha_n, \beta_n) \) is the \( \langle \text{lex} \rangle \)-least \( (\forall \varphi, \alpha, \beta) \) such that there is some \( y \in \omega \) such that
(a) \( \{ u \in V_\alpha : V_\alpha \models \varphi(u, \beta, y) \} \in a \), and
(b) if \( n > 0 \), then \( (x_{n-1}, \{ u \in V_\alpha : V_\alpha \models \varphi(u, \beta, y) \}) \in R \)
and \( x_n \in \omega \) is such that
(a) \( \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n) \} \in a \), and
(b) if \( n > 0 \), then \( (x_{n-1}, \{ u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n) \}) \in R \).
Let \( x = \oplus_{n<\omega} x_n \). It is then easy to see that \( f \) is definable in \( V[G] \) from \( x \) together with an ordinal and a real parameter which defines \( (a,R) \). Hence \( f \in N \).

p.170 l.14: replace \( C \cap U_s \cap \mathcal{O}^s_n = \emptyset \) by \( C \cap U_s \cap \mathcal{O}^s_n \neq \emptyset \). (Thanks to Vincenzo Dimonte!)

p.180 l.2 from b.: replace \( g(n) \leq m < g(n+1) \) by \( f(n) \leq m < f(n+1) \)

p.189 l.10: replace \( \lceil Y \rceil^{<\omega} \) by \( \lceil Y \rceil^n \)

p.193: the 2nd displayed formula should read \( h : \bigcup_{n<\omega} \{1\} \times v(n)M \to M \)

p.195 l.6 from b.: replace \( \{ \xi < \kappa : f(x) \in y \} \) by \( \{ \xi < \kappa : f(\xi) \in y \} \). (Thanks to Vincenzo Dimonte!)

p.199 l.7 from b.: replace “Lemma 1.31 (g) and (e)” by “Lemma 10.21 (g) and (e).” (Thanks to Vincenzo Dimonte!)

p.199 l.2 from b.: replace “Lemma 10.29 (h)” by “Lemma 10.21 (h).” (Thanks to Vincenzo Dimonte!)

p.203 l.3 from b.: replace “\( \text{In (c)} \)” by “\( \text{In (iii)} \)” (Thanks to Vincenzo Dimonte!)

p.210 Definition 10.45: It has to be added that if \( E \) is a \( (\kappa, \nu) \)-extender, then \( \nu \) is called the length of \( E \). The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)
p.212 l.5f.: \( g : [\mu_\theta]^{\text{Card}(b)} \to M \)

p.226: \( U^* \) refers to two different things on this page, to a tree, defined l.9, and to a substructure of \( R_i \), defined l.17 (display). Also, \( \tau \) refers to two different things on this page, to \( \sigma' \mid V_{\nu_i}^M \), defined l.9, and to a map from (the 2nd) \( U^* \) to \( V_\kappa^R \), defined l.18. There is also a sloppyness about \( \Sigma_{1+} \) formulae on this page in that the first parameter (free variable) of \( \Phi \) got suppressed: e.g. in (10.46) by \( \Phi(\sigma_i \mid V_{\nu_i}^M) \) I really meant \( \Phi(\sigma_i(\nu_i), \sigma_i \mid V_{\nu_i}^M) \), i.e., \( \Phi(\tau) \) in l.7 should have been written as \( \Phi(\tau(\nu_i), \tau) \) – with the understanding that \( \tau(\nu_i) = \sup(\tau^{\nu_i}) \). (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make \( \forall x \in U' \exists y \in U' x \in y \) part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take \( U = 4, U' = 4 \cup \{\{0, 2\}\} \), and \( \pi = \text{id} \). (Thanks to Toby Meadows!)

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.

p.280 l.8: \( z \in \omega X \)

p.280: the displayed formula in the middle should read \( \{z \ast \tau : z \in \omega X\} \subset \omega \setminus A \)

p.282 l.1: \( s = (x_0, \ldots, x_{n-1}) \)

p.306 l.-12: replace \( \{\pi_{s,x}(\kappa_n) : n < \omega, s \subset x, \text{lh}(s) = n + 1\} \) by \( \{\pi_{s,x}(\kappa_n) : n < \omega, s \subset x, \text{lh}(s) \geq n + 1\} \). (Thanks to Andreas Lietz!)