Errata of the book

“Set theory. Exploring Independence and Truth”

by Ralf Schindler

p.2 l.-6: delete the first “u” in “analogous.”

p.4, l.8f.: the definition should read: “A set \( B \subseteq A \) is called dense in \( A \) iff for all \( a, b \in A \) with \( a < b \) and \( (a, b) \cap A \neq \emptyset \), then \( (a, b) \cap B \neq \emptyset \). (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read “Proof of Theorem 1.9,” not “Proof of Theorem 1.8.”

p.5 l.16: delete the last “that.”

paragraph at the bottom of p.6 and the top of p.7: delete the sentence “As \( \mathcal{Q} \) is dense [...] picked to be pairwise disjoint.”

p.8 l.-2: delete “[\( a, b \]_\infty is dense in [\( a, b \])].” This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that \( b \) does not have a maximum [...].

p.18 l.-7: delete “the.”

p.20 l.10: Shat that [...]

p.23 l.8: insert “is” before “inductive.”

p.27 l.9: replace “the \( R \)-least \( x_0 \)” by “an \( R \)-least \( x_0 \).” (Thanks to Philipp Schlicht!)

p.34 l.1: “my” should be “may.”

p.35 l.3f.: ... for cardinals \( \kappa, \lambda \) with \( \lambda \leq \kappa \).

p.35 l.20: replace \( \pi(\gamma) \) by \( \pi((\gamma, \gamma)) \). Similarly, l.25: replace \( \pi(\aleph_0) \) by \( \pi((\aleph_0, \aleph_0)) \), l.27: relace \( \pi(\aleph_0) \) by \( \pi((\aleph_0, \aleph_0)) \).

p.37 l.16: replace “is” by “in.”

p.38 l.7: replace “Poblem” by “Problem.”

p.41 l.-3: replace \( A_{\kappa \eta} \) by \( A_{\eta} \).

p.43 l.1: replace “from” by “form.”

p.43 l.2: replace \( \kappa^+ \) by \( \kappa \).

p.44 l.3: replace \( \gamma^\alpha \mathcal{S} \) by \( \gamma^\alpha \in \mathcal{S} \).

p.44 l.-10: replace \( g_i: [\mu_i]^\kappa(\kappa) \rightarrow \mu_i^+ \) by \( g_i: [\mu_i]^{\leq \text{cl}(\kappa)} \rightarrow \mu_i^+ \).

p.44 l.-3: unfortunately, this is not the same \( g_i \) as in l.-10 of the same page.

p.45 l.2: delete “in.”
p.45 l.11 and l.15: replace \( P(\kappa) \) by \([\kappa]^{\text{cf}(\kappa)}\); also lines 11, 14, 15, and 16: replace \( Y \subset \kappa \) by \( Y \in [\kappa]^{\text{cf}(\kappa)} \).

p.62, Problem 4.4: cf. p.35 l.3f.

p.97 footnote 1: replace “until p. 97” by “until p. 101.”

p.139 l.7 from b.: This should say “Also, if \((2^{\aleph_0})^{V[x]} = \omega_1^{L[x]} < 2^{\aleph_0}\), then by Lemmas 7.19 and 7.20 there is a largest \( \Sigma^1_2(x) \)-set of reals which is smaller than \( 2^{\aleph_0} \), namely \( \omega \omega \cap L[x] \).”

p.140: the 2nd last displayed formula on that page got screwd up. The aim is to choose \( x \) such that \( \langle \varphi_0(x), x(0), \varphi_1(x), x(1), \ldots \rangle \) is \(<_{\text{lex}}\)-minimal. Let the formula read:

\[
[x \mid n = y \mid n \land \forall m < n(\varphi_m(x) = \varphi_m(y))] \rightarrow \\
[y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y) \lor (\varphi_n(x) = \varphi(y) \land x(n) \leq y(n)))).
\]

As explained in the text, for \( x \in A \) “\( y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y))\)” and “\( y \notin A \lor (y \in A \land (\varphi_n(x) \leq \varphi_n(y)))\)” can both be uniformly written in a \( \Pi^1_1 \) as well as in a \( \Sigma^1_1 \) way, so that the relevant formula is \( \Pi^1_1 \). (Thanks to Robin Puchalla!)

p.210 Definition 10.45: It has to be added that if \( E \) is a \((\kappa, \nu)\)-extender, then \( \nu \) is called the length of \( E \). The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)

p.226: \( U^* \) refers to two different things on this page, to a tree, defined l.9, and to a substructure of \( R_\varsigma \), defined l.17 (display). Also, \( \tau \) refers to two different things on this page, to \( \sigma' \upharpoonright V_{\nu_i}^{M_i} \), defined l.9, and to a map from (the 2nd) \( U^* \) to \( V_{\nu_i}^{R_i} \), defined l.18. There is also a sloppyness about \( \Sigma^1_\frac{1}{2} \) formulae on this page in that the first parameter (free variable) of \( \Phi \) got suppressed: e.g. in (10.46) by \( \Phi(\sigma_{i \upharpoonright V_{\nu_i}^{M_i}}) \) I really meant \( \Phi(\sigma_i(\nu_i), \sigma_{i \upharpoonright V_{\nu_i}^{M_i}}, \tau) \), i.e., \( \Phi(\tau) \) in l.7 should have been written as \( \Phi(\tau(\nu_i), \tau) \) – with the understanding that \( \tau(\nu_i) = \sup(\tau \upharpoonright \nu_i) \). (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make “\( \forall x \in U' \exists y \in U' \ x \in y' \) part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take \( U = 4, U' = 4 \cup \{0, 2\} \), and \( \pi = \text{id.} \). (Thanks to Toby Meadows!)

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.