Errata of the book

“Set theory. Exploring Independence and Truth”

by Ralf Schindler

p.2 l.-6: delete the first “u” in “analoguous.”

p.4, l.8f.: the definition should read: “A set $B \subset A$ is called dense in $A$ iff for all $a, b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read “Proof of Theorem 1.9,” not “Proof of Theorem 1.8.”

p.5 l.16: delete the last “that.”

paragraph at the bottom of p.6 and the top of p.7: delete the sentence “As $Q$ is dense [...] picked to be pairwise disjoint.”

p.8 l.-2: delete “[a,b]_\infty$ is dense in [a,b].” This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that $b$ does not have a maximum [...].

p.18 l.-7: delete “the.”

p.20 l.10: Shat that [...]

p.23 l.8: insert “is” before “inductive.”

p.27 l.9: replace “the $R$–least $x_0$” by “an $R$–least $x_0$.” (Thanks to Philipp Schlicht!)

p.34 l.1: “my” should be “may.”

p.35 l.3f.: ... for cardinals $\kappa, \lambda$ with $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(R_0)$ by $\pi((R_0, R_0))$, l.27: replace $\pi(R_0)$ by $\pi((R_0, R_0))$.

p.37 l.16: replace “is” by “in.”

p.38 l.7: replace “Poblem” by “Problem.”

p.41 l.-3: replace $A_{\kappa\eta}$ by $A_{\eta\kappa}$.

p.43 l.1: replace “from” by ”form.”

p.43 l.2: replace $\kappa^+$ by $\kappa$.

p.44 l.3: replace $\gamma^\alpha_{\eta'} S$ by $\gamma^\alpha_{\eta'} \in S$.

p.44 l.-10: replace $g_i : [\mu_i]^{\delta(\kappa)} \rightarrow \mu_i^+$ by $g_i : [\mu_i]^{\leq \delta(\kappa)} \rightarrow \mu_i^+$. p.44 l.-3: unfortunately, this is not the same $g_i$ as in l.-10 of the same page.

p.45 l.2: delete “in.”
p.45 l.11 and l.15: replace \( \mathcal{P}(\kappa) \) by \([\kappa]^{\text{cf}(\kappa)}\); also lines 11, 14, 15, and 16: replace \( Y \subset \kappa \) by \( Y \in [\kappa]^{\text{cf}(\kappa)}\).

p.49f.: The construction of a \( \kappa^+ \) Aronszajn tree is imprecise. Let us fix it as follows.

Let \( \{A_s: s \in ^{<\omega}\kappa\} \) be such that \( A_0 = \kappa \) and for all \( s \in ^{<\omega}\kappa \), \( \{A_s: \xi < \kappa\} \) is a family of pairwise disjoint sets with \( A_s = \bigcup\{A_t: \xi < \kappa\} \). Let \( A = \{\cup B: B \in \{A_s: s \in ^{<\omega}\kappa \land s \neq \emptyset\}\}^{<\kappa} \).

By \( \kappa^{<\kappa} = \kappa \), \( \text{Card}(A) = \kappa \).

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all \( s \in T \) there is some \( A \in \mathcal{A} \) with \( \text{ran}(s) \subset A \).

(2) If \( s \in T \), \( \text{ran}(s) \subset A, B \in \mathcal{A}, B \cap A = \emptyset \), \( \text{lv}_T(s) < \beta < \kappa^+ \), then there is some \( t \in T \) with \( \text{lv}_T(t) = \beta, s \subset t \), and \( \text{ran}(t) \subset \text{ran}(s) \cup B \).

(4) Let \( \lambda < \kappa^+ \) be a limit ordinal with \( \text{cf}(\lambda) < \kappa \). Let \( C \subset \lambda \) be club in \( \lambda \) with \( \text{otp}(C) = \text{cf}(\lambda) \), and let \( \{\lambda_i: i < \text{cf}(\lambda)\} \) be the monotone enumeration of \( \{0\} \cup C \). Let \( \{A_i: i < \text{cf}(\lambda)\} \cup \{B\} \) be a pairwise disjoint family of elements of \( \mathcal{A} \). Let \( s: \lambda \to \kappa \) be such that \( s \upharpoonright \lambda_i = \lambda_i \cup s_i^\kappa \lambda_i \subset \bigcup\{A_j: j < i\} \) for every \( i < \text{cf}(\lambda) \). Then \( s \in T_{\lambda+1} \).

The rest is as before except that in case \( \text{cf}(\lambda) = \kappa \) we pick \( s(t) \) as follows. We fix \( C \subset \lambda \), a club in \( \lambda \) with \( \text{otp}(C) = \kappa \), and we let \( \{\lambda_i: i < \kappa\} \) be the monotone enumeration of \( \{0\} \cup C \). By the new (1), \( \text{ran}(s) \subset A \in \mathcal{A} \) for some \( A \). Let \( \{A_i: i < \text{cf}(\lambda)\} \cup \{B\} \) be such that \( \{A_i: i < \text{cf}(\lambda)\} \cup \{A, B\} \) is a pairwise disjoint family of elements of \( \mathcal{A} \). (This choice is possible!) Using the new (2) and the new (4), we may construct some \( t: \lambda \to \kappa \) extending \( s \) such that for every \( i < \kappa \),

\[
t \upharpoonright \lambda_i = T_{\lambda+1} \land t^\kappa \lambda_i \subset A \cup \bigcup\{A_j: j < i\}.
\]

We write \( t(s) \) for this \( t \). Then we let \( T_{\lambda+1} = T_\lambda \cup \{t(s): s \in T_\lambda\} \).

p.62, Problem 4.4: cf. p.35 l.3f.

p.97 footnote 1: replace “until p. 97” by “until p. 101.”

p.106 l.19: add “, and \( \{\xi_k^p: p \in D_1\} \) is unbounded in \( \omega_1 \).” (Let \( \beta \in \omega_1 \) be such that \( \xi_k^p < \beta \) for all \( p \in D_0 \) and \( 1 \leq k < k_0 \). If for all \( \xi_1 < \ldots < \xi_{k_0-1} \leq \beta \) and for all \( s_1, \ldots, s_{k_0-1} \in ^{<\omega}\omega \), the unique \( \{\xi_k^p: p \in D_0 \land \xi_k^p = \xi_j \land \ldots \land \xi_{k_0-1}^p = \xi_{k_0-1} \land p(\xi_1) = s_1 \land \ldots \land p(\xi_{k_0-1}) = s_{k_0-1}\} \) bounded in \( \omega_1 \), then there would be one common bound for all \( \xi_1 < \ldots < \xi_{k_0-1} \leq \beta \) and \( s_1, \ldots, s_{k_0-1} \in ^{<\omega}\omega \), contradicting the choice of \( k_0 \).) In l.23f., replace “By the choice of \( k_0 \)” with “By the choice of \( D_1 \).”

p.139 l.7 from b.: This should say “Also, if \( 2^{2^{k_0}} = \omega_1^{1^{[x]} = 2^{k_0}} \), then by Lemmas 7.19 and 7.20 there is a largest \( \Sigma^1_4(x) \)-set of reals which is smaller than \( 2^{2^{k_0}} \), namely \( \omega_1 \land L[x] \).”

p.140: the 2nd last displayed formula on that page got screwd up. The aim is to choose \( x \) such that \( \langle \varphi_0(x), x(0), \varphi_1(x), x(1), \ldots \rangle \) is \( <_{\text{lex}} \)-minimal. Let the formula read:

\[
[x \mid n = y \mid n \land \forall m < n(\varphi_m(x) = \varphi_m(y)) \to
\]
\[y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y)) \lor (\varphi_n(x) = \varphi(y) \land x(n) \leq y(n)))\].

As explained in the text, for \(x \in A\), "\(y \notin A \lor (y \in A \land (\varphi_n(x) < \varphi_n(y)))\)" and "\(y \notin A \lor (y \in A \land (\varphi_n(x) \leq \varphi_n(y)))\)" can both be uniformly written in a \(\Pi_1\) as well as in a \(\Sigma_1\) way, so that the relevant formula is \(\Pi_1\). (Thanks to Robin Puchalla!)

p.210 Definition 10.45: It has to be added that if \(E\) is a \((\kappa, \nu)\)-extender, then \(\nu\) is called the length of \(E\). The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)

p.226: \(U^*\) refers to two different things on this page, to a tree, defined l.9, and to a substructure of \(R_i\), defined l.17 (display). Also, \(\tau\) refers to two different things on this page, to \(\sigma' \restriction V_{\nu_i}^M\), defined l.9, and to a map from (the 2nd) \(U^*\) to \(V_{R_i}^\kappa\), defined l.18. There is also a sloppyness about \(\Sigma_1^+\) formulae on this page in that the first parameter (free variable) of \(\Phi\) got suppressed: e.g. in (10.46) by \(\Phi(\sigma_i \restriction V_{\nu_i}^M)\) I really meant \(\Phi(\sigma_i(\nu_i), \sigma_i \restriction V_{\nu_i}^M)\), i.e., \(\Phi(\tau)\) in l.7 should have been written as \(\Phi(\tau(\nu_i), \tau)\) – with the understanding that \(\tau(\nu_i) = sup(\tau^\nu i)\). (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make \("\forall x \in U' \exists y \in U' \ x \in y"\) part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take \(U = 4, U' = 4 \cup \{(0, 2)\}\), and \(\pi = id\). (Thanks to Toby Meadows!)

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.