Errata of the book
"Set theory. Exploring Independence and Truth"
by Ralf Schindler

I cordially thank Vincenzo Dimonte, Fan Feng, Stefan Hoffelner, Milad Khodayi, Jan Kruschewski, Andreas Lietz, Bob Lubarsky, Toby Meadows, Alexander Paseau, Robin Puchalla, Philipp Schlicht, Shervin Sorouri, and Xiuyuan Sun for pointing out errors/typos to me.

p.2 l.-6: delete the first "u" in "analoguous."

p.4, l.8f.: the definition should read: "A set $B \subset A$ is called *dense in* A iff for all a, $b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read "Proof of Theorem 1.9," not "Proof of Theorem 1.8."

p.5 l.16: delete the last "that."

paragraph at the bootom of p.6 and the top of p.7: delete the sentence "As $\mathbb{Q}$ is dense [...] picked to be pairwise disjoint."

p.8 l.-2: delete "$[a, b]_\infty$ is dense in $[a, b]$." This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that b does *not* have a maximum [...].

p.18 l.-7: delete "the."

p.20 l.10: Shat *that* [...]

p.23 l.8: insert "is" before "inductive."

p.27 l.9: replace "the R–least x_0" by "an R–least x_0." (Thanks to Philipp Schlicht!)

p.34 l.1: "my" should be "may."

p.35 l.3f.: ... for cardinals κ, λ *with* $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(\aleph_0)$ by $\pi((\aleph_0, \aleph_0))$, l.27: relace $\pi(\aleph_\alpha)$ by $\pi((\aleph_\alpha, \aleph_\alpha))$.

p.37 l.16: replace "is" by "in."

p.38 l.7: replace "Poblem" by "Problem."

p.41 l.-3: replace $A_{x\eta_n}$ by A_{η_n}.

p.43 l.1: replace "from" by "form."

p.43 l.2: replace κ^+ by κ.

p.44 l.3: replace $\gamma_\alpha^{\alpha'} \bar{S}$ by $\gamma_\alpha^{\alpha'} \in \bar{S}$.

p.44 l.-10: replace $g_i \colon [\mu_i]^{\mathrm{cf}(\kappa)} \to \mu_i^+$ by $g_i \colon [\mu_i]^{\leq \mathrm{cf}(\kappa)} \to \mu_i^+$.

p.44 l.-3: unfortunately, this is not the same g_i as in l.-10 of the same page.

p.45 l.2: delete "in."

p.45 l.11 and l.15: replace $\mathcal{P}(\kappa)$ by $[\kappa]^{\mathrm{cf}(\kappa)}$; also lines 11, 14, 15, and 16: replace $Y \subset \kappa$ by $Y \in [\kappa]^{\mathrm{cf}(\kappa)}$.

p.47 Definition 4.44 (3): replace "$\mathrm{ht}(T) = \alpha$" by "$\mathrm{ht}(t) = \alpha$."

p.48 l.4: replace (2) by (4).

p.49f.: The construction of a κ^+ Aronszajn tree is imprecise. Let us fix it as follows.

Let $(A_s \colon s \in {}^{<\omega}\kappa)$ be such that $A_\emptyset = \kappa$ and for all $s \in {}^{<\omega}\kappa$, $\{A_{s^\frown\xi} \colon \xi < \kappa\}$ is a family of pairwise disjoint sets with $A_s = \bigcup\{A_{s^\frown\xi} \colon \xi < \kappa\}$. Let

$$\mathcal{A} = \{\cup B \colon B \in [\{A_s \colon s \in {}^{<\omega}\kappa \wedge s \neq \emptyset\}]^{<\kappa}\}.$$

By $\kappa^{<\kappa} = \kappa$, $\mathrm{Card}(\mathcal{A}) = \kappa$.

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all $s \in T$ there is some $A \in \mathcal{A}$ with $\mathrm{ran}(s) \subset A$.

(2) If $s \in T$, $\mathrm{ran}(s) \subset A \in \mathcal{A}$, $B \in \mathcal{A}$, $B \cap A = \emptyset$, $\mathrm{lv}_T(s) < \beta < \kappa^+$, then there is some $t \in T$ with $\mathrm{lv}_T(t) = \beta$, $s \subset t$, and $\mathrm{ran}(t) \subset \mathrm{ran}(s) \cup B$.

(4) Let $\lambda < \kappa^+$ be a limit ordinal with $\mathrm{cf}(\lambda) < \kappa$. Let $C \subset \lambda$ be club in λ with $\mathrm{otp}(C) = \mathrm{cf}(\lambda)$, and let $(\lambda_i \colon i < \mathrm{cf}(\lambda))$ be the monotone enumeration of $\{0\} \cup C$. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be a pairwise disjoint family of elements of $\mathcal{A}$. Let $s \colon \lambda \to \kappa$ be such that

$$s \upharpoonright \lambda_i \in T_{\lambda_i+1} \wedge s''\lambda_i \subset \bigcup\{A_j \colon j < i\}$$

for every $i < \mathrm{cf}(\lambda)$. Then $s \in T_{\lambda+1}$.

The rest is as before except that in case $\mathrm{cf}(\lambda) = \kappa$ we pick $s(t)$ as follows. We fix $C \subset \lambda$, a club in λ with $\mathrm{otp}(C) = \kappa$, and we let $(\lambda_i \colon i < \kappa)$ be the monotone enumeration of $\{0\} \cup C$. By the new (1), $\mathrm{ran}(s) \subset A \in \mathcal{A}$ for some A. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be such that $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{A, B\}$ is a pairwise disjoint family of elements of $\mathcal{A}$. (This choice is possible!) Using the new (2) and the new (4), we may construct some $t \colon \lambda \to \kappa$ extending s such that for every $i < \kappa$,

$$t \upharpoonright \lambda_i \in T_{\lambda_i+1} \wedge t''\lambda_i \subset A \cup \bigcup\{A_j \colon j < i\}.$$

We write $t(s)$ for this t. We then let $T_{\lambda+1} = T_\lambda \cup \{t(s) \colon s \in T_\lambda\}$.

p.54 l.8: $\{\xi < \kappa \colon f(\xi) \in g(\xi)\} \in U$

p.56 l.4: Theorem

p.60: In the proof of the weak normality of U, all occurences of κ have to be replaced by λ.

p.62, Problem 4.4: cf. p.35 l.3f.

p.64, Problem 4.23: In (b), require that H be a model of ZFC^-.

p.64, Problem 4.25: This is wrong for trivial reasons. The statement has to be adjusted as follows. If κ is ineffable, then there is no slim κ-KUREPA tree. Here, a κ-tree T is *slim* iff for all $\alpha < \kappa$, $T_\alpha = \{s \in T\colon \mathrm{lv}_T(s) < \alpha\}$ has size at most $\mathrm{Card}(\alpha)$. (Thanks to Shervin Sorouri!)

p.68 Lemma 5.5: we also need to assume that T is a subtheory of ZFC or at least that $V \models T$.

pp.72f. and p.88: The proof of Lemma 5.11 is wrong, as not every rud_E function is simple in the sense of the definition of "simple" given in "$\subset$" of that proof. E.g. if $f(\vec{x})$ is rud_E, then the formula $f(\vec{x}) \in E$ need not be Σ_0 in the language $\mathcal{L}_{\in,E}$. (Thanks to Shervin Sorouri!)

This proof should be fixed as follows. First,

$$\mathcal{P}(U) \cap \underset{\sim}{\Sigma}_\omega(U; \in, E) = \mathcal{P}(U) \cap \underset{\sim}{\Sigma}_0(U \cup \{U, E\}; \in),$$

so that we have to prove that

$$\mathcal{P}(U) \cap \mathrm{rud}_E(U \cup \{U\}) = \mathcal{P}(U) \cap \underset{\sim}{\Sigma}_0(U \cup \{U, E\}; \in).$$

In the proof of "$\subset$," the definition of "simple" then has to be adjusted as follows: Let a formula φ in $\mathcal{L}_{\in,E}$ be Σ_0' iff φ is in the smallest class Γ of formulas such that

(a) all atomic formulas are in Γ,

(b) Γ is closed under sentential connectives,

(c) Γ is closed under bounded quantification, and

(d) if ψ is in Γ and x is a variable, then $\exists x \in E\, \psi$ and $\forall x \in E\, \psi$ are both in Γ.

These are the closure conditions as in Definition 5.1 on p.67 *plus* that quantification over elements of E counts as bounded quantification.

Now call a function $f\colon V^k \to V$, where $k < \omega$, *simple* iff the following holds true: if $\varphi(v_0, v_1, \ldots, v_m)$ is Σ_0' in the language $\mathcal{L}_{\in,E}$, then $\varphi(f(v_1', \ldots, v_k'), v_1, \ldots, v_m)$ is equivalent over rud_E closed structures to a Σ_0' formula in the same language.

This is also the definition of "simple" which should be used in Problem 5.8 on p.88.

p.75 l.10: the indices β in should of course all be limit ordinals.

p.76: the definition of F_{14} should be $F_{14} = \{(x, (y)_0), (y)_1\}$.

p.76 last line before the statement of Lemma 5.20: replace "Problem 5.5" by "Problem 5.6."

p.84 l.7 from bottom: In the definition of D, also require $\kappa \in g"\xi$.

p.85: In the proof of Lemma 5.40, we should also assume that every $\xi \in S$ is closed under GÖDEL pairing.

p.86 l.5 from bottom: cross out one of the "is"

p.87 l.14 from bottom: replace $\leq^*$ by $\leq^*_z$. l.9 from bottom: the 2nd β in the displayed formula should be boldface. Last line: replace $\leq^{**}$ by $\leq^{***}$.

p.88 Problem 5.4: κ must also be assumed to be uncountable.

p.97 footnote 1: replace "until p. 97" by "until p. 101."

p.106 l.19: add ", and $\{\xi^p_{k_0} : p \in D_1\}$ is unbounded in ω_1." (Let $\beta < \omega_1$ be such that $\xi^p_k < \beta$ for all $p \in D_0$ and $1 \le k < k_0$. If for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and for all $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$ the set $\{\xi^p_{k_0} : p \in D_0 \wedge \xi^p_1 = \xi_1 \wedge \ldots \wedge \xi^p_{k_0-1} = \xi_{k_0-1} \wedge p(\xi_1) = s_1 \wedge \ldots \wedge p(\xi_{k_0-1}) = s_{k_0-1}\}$ were bounded in ω_1, then there would be one common bound for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$, contradicting the choice of k_0.) In l.23f., replace "By the choice of k_0" with "By the choice of D_1."

p.109 proof of Lemma 6.38: $\tau^G = f$

p.110 l.2 from b.: replace $\mu = \cdot 2^{<\mu}$ by $\mu = 2^{<\mu}$

p.118 Corollary 6.62: Add "$\mathbb{P}$ is separative" to the hypotheses in the statement of this corollary, as its proof makes use of Lemma 6.60. (Thanks to Fan Feng!)

p.120 l.5 from b.: replace "Lemma 6.65" by "Lemma 6.32." (Thanks to Fan Feng!)

p.131, last line of the proof of lemma 7.8: replace S by S_x.

p.135 l.4: the first occurance of "$x \in B$" is to be replaced by "$x \notin B$.

p.137 l.5: both occurances of T^∞ are to be replaced by $T^\infty_{(s_u,t_u)}$.

p.137f., proof of Claim 7.18: The construction of U is wrong. Let $\delta, \epsilon \mapsto \langle \delta, \epsilon \rangle$ be the GÖDEL pairing function, see p.35. Then let $(s,t) \in U$ iff for all $n < \omega$, $(s \restriction k, n, (t(\langle n, 0\rangle), \ldots, t(\langle n, k-1\rangle))) \in U$, where $k \ge 0$ is maximal such that $k = 0$ or $\langle n, k-1\rangle < \mathrm{lh}(t) = \mathrm{lh}(s)$. (Thanks to Andreas Lietz and Stefan Hoffelner!)

p.138 l.14: Π_n has to be replaced by Σ_n.

p.139 l.7 from b.: This should say "Also, if $(2^{\aleph_0})^{L[x]} = \omega_1^{L[x]} < 2^{\aleph_0}$, then by Lemmas 7.19 and 7.20 there is a *largest* $\Sigma^1_2(x)$–set of reals which is smaller than $2^{\aleph_0}$, namely ${}^\omega\omega \cap L[x]$."

p.139 l.-3: in Definition 7.22, $F(y)$ has to be replaced by $F(x)$.

p.140: the 2nd last displayed formula on that page got screwd up. The aim is to choose x such that $(\varphi_0(x), x(0), \varphi_1(x), x(1), \ldots)$ is $<_{\mathrm{lex}}$–minimal. Let the formula read:

$$[x \restriction n = y \restriction n \wedge \forall m < n(\varphi_m(x) = \varphi_m(y))] \longrightarrow$$
$$[y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y) \vee (\varphi_n(x) = \varphi(y) \wedge x(n) \le y(n)))].$$

As explained in the text, for $x \in A$, "$y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y)))$" and "$y \notin A \vee (y \in A \wedge (\varphi_n(x) \le \varphi_n(y)))$" can both be uniformly written in a Π^1_1 as well as in a Σ^1_1 way, so that the relevant formula is Π^1_1. (Thanks to Robin Puchalla!)

p.162 last two lines: The proof of $N \models \mathsf{DC}$ has to be fixed as follows. Let R, $a \in N$ be such that for all $x \in a$ there is a $y \in a$ with $(x, y) \in R$. Let us work in $V[G]$ and define $f : \omega \to a$ as follows. For each $n < \omega$,

$$f(n) = \{u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi_n(u, \beta_n, x_n)\},$$

where $(\ulcorner\varphi_n\urcorner, \alpha_n, \beta_n)$ is the $<_{\mathrm{lex}}$-least $(\ulcorner\varphi\urcorner, \alpha, \beta)$ such that there is some $y \in {}^\omega\omega$ such that

(a) $\{u \in V_\alpha \colon V_\alpha \models \varphi(u, \beta, y)\} \in a$, and

(b) if $n > 0$, then $(x_{n-1}, \{u \in V_\alpha \colon V_\alpha \models \varphi(u, \beta, y)\}) \in R$

and $x_n \in {}^\omega\omega$ is such that

(a) $\{u \in V_{\alpha_n} \colon V_{\alpha_n} \models \varphi(u, \beta_n, x_n)\} \in a$, and

(b) if $n > 0$, then $(x_{n-1}, \{u \in V_{\alpha_n} \colon V_{\alpha_n} \models \varphi(u, \beta_n, x_n)\}) \in R$.

Let $x = \oplus_{n<\omega} x_n$. It is then easy to see that f is definable in $V[G]$ from x together with an ordinal and a real parameter which defines (a, R). Hence $f \in N$.

p.170 l.14: replace $C \cap U_s \cap \mathcal{O}_n^x = \emptyset$ by $C \cap U_s \cap \mathcal{O}_n^x \neq \emptyset$. (Thanks to Vincenzo Dimonte!)

p.180 l.2 from b.: replace $g(n) \leq m < g(n+1)$ by $f(n) \leq m < f(n+1)$

p.187 l.9 from b.: replace $f_k(\kappa_{\bar\alpha})$ by $\pi_{\bar\alpha,\alpha}(f_k)(\kappa_{\bar\alpha})$

p.189 l.10: replace $[Y]^{<\omega}$ by $[Y]^n$

p.193: the 2nd displayed formula should read $h \colon \bigcup_{n<\omega}(\{n\} \times {}^{v(n)}M) \to M$. l.10 from b.: replace $\neg(j, \mathsf{b}) \in \models_M^{\Sigma_0}$ by $\neg(j, \bar{\mathsf{b}}) \in \models_M^{\Sigma_0}$

p.195 l.6 from b.: replace $\{\xi < \kappa \colon f(x) \in y\}$ by $\{\xi < \kappa \colon f(\xi) \in y\}$. (Thanks to Vincenzo Dimonte!)

p.199 l.7 from b.: replace "Lemma 1.31 (g) and (e)" by "Lemma 10.21 (g) and (e)." (Thanks to Vincenzo Dimonte!)

p.199 l.2 from b.: replace "Lemma 10.29 (h)" by "Lemma 10.21 (h)." (Thanks to Vincenzo Dimonte!)

p.203 l.3 from b.: replace "In (c)" by "In (iii)" (Thanks to Vincenzo Dimonte!)

p.210 Definition 10.45: It has to be added that if E is a (κ, ν)–extender, then ν is called the *length of* E. The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)

p.212 l.5f.: $g \colon [\mu_b]^{\mathrm{Card}(b)} \to M$

p.226: U^* refers to two different things on this page, to a tree, defined l.9, and to a substructure of R_{i_2} defined l.17 (display). Also, τ refers to two different things on this page, to $\sigma' \restriction V_{\nu_i}^{\bar M_i}$, defined l.9, and to a map from (the 2nd) U^* to $V_\kappa^{R_i}$, defined l.18. There is also a sloppyness about Σ_{1+} formulae on this page in that the first parameter (free variable) of Φ got suppressed: e.g. in (10.46) by $\Phi(\sigma_i \restriction V_{\nu_i}^{\bar M_i})$ I really meant $\Phi(\sigma_i(\nu_i), \sigma_i \restriction V_{\nu_i}^{\bar M_i})$, i.e., $\Phi(\tau)$ in l.7 should have been written as $\Phi(\tau(\nu_i), \tau)$ – with the understanding that $\tau(\nu_i) = \sup(\tau"\nu_i)$. (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make "$\forall x \in U' \, \exists y \in U' \, x \in y$" part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take $U = 4$, $U' = 4 \cup \{\{0, 2\}\}$, and $\pi = \mathrm{id}$. (Thanks to Toby Meadows!)

p.240 1st displayed formula in the proof of Lemma 11.16: $\pi(\xi)$ should be $\pi(\mathbf{z})$.

p.241 l.10 from bottom: $[\rho_1(\bar M)]^{\geq\omega}$ should be $[\rho_1(\bar M)]^{<\omega}$.

p.245 middle: in the definition of $[n, x]$, $[m, y]$ needs to be replaced by (m, y).

p.246 upper half middle: in the definition of $[n, x]^*$, $[m, y]$ needs to be replaced by (m, y).

p.249 l.9 from bottom: delete "$r l$" at the beginning of the displayed formula.

p.250 l.2 from bottom: in the definition of Γ_M^{n+1}, the superscript $\geq \omega$ needs to be replaced by $< \omega$.

p.252 l.11 from bottom (displayed formula): $h_{M''}^{n+1,p}(\omega \times {}^{<\omega}X)$ needs to be replaced by $h_M^{n+1,p}$ "$(\omega \times {}^{<\omega}X)$.

p.255 l.17: replace $q(n)$ by $q(n-1)$.

p.257 proof of Lemma 11.43: There should be a case split in this proof, as the proof which is written down implicitly assumes that $\nu \in W_M^{\nu,p}$. If $\nu = \max(p)$, then it might be possible that $\nu = W_M^{\nu,p} \cap \mathrm{OR}$, in which case $W_M^{\nu,p} = M|\nu \in M$ is trivial, though. (Thanks to Andreas Lietz!)

p.257 l.5-4 from bottom: "$\sigma(\nu)$ is regular in M" needs to be replaced by "$\tau(\nu)$ is regular in M." Also, l.2-1 from bottom: all three occurances of $J_{\sigma(\nu)}[B]$ should be replaced by $J_{\tau(\nu)}[E']$.

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.

p.280 l.8: $z \in {}^\omega X$. l.10: replace σ by τ

p.280: the displayed formula in the middle should read $\{z * \tau \colon z \in {}^\omega X\} \subset {}^\omega\omega \setminus A$

p.280 last l.: replace σ_α by τ_α

p.281 l.3: replace $z * \tau_\alpha * z$ by $z * \tau_\alpha$

p.281, in the displayed table after l.7, replace x_3 by n_3

p.282 l.1: $s = (x_0, \ldots, x_{n-1})$

p.285 l.9 from b.: replace $s_0{}^\frown n_0{}^\frown \ldots {}^\frown s_i{}^\frown n_i$ by $s_0{}^\frown n_0{}^\frown \ldots {}^\frown s_k{}^\frown n_k$. lines 7 and 4 from b.: replace $s \in {}^\omega 2$ by $s \in {}^{<\omega}2$

p.287, displayed formula on the top: replace $s \in {}^{<\omega}2 \setminus \emptyset$ by $s \in {}^{<\omega}2 \setminus \{\emptyset\}$

p.289, in the proof of (b) replace "a cone for a base" by "a base for a cone"

p.295 l.17 from b.: replace $s' \in {}^\omega\omega$ by $s' \in {}^{<\omega}\omega$

p.300 l.4: replace $b \in J_\alpha[\tau]$ by $b \in J_\alpha[x, \tau]$

p.305 l.5: replace $\alpha_{s\restriction i+1}(x) < \pi_{s\restriction i, s\restriction i+1}(x)$ by $\alpha_{s\restriction i+1}(x) < \pi_{s\restriction i, s\restriction i+1}(\alpha_{s\restriction i}(x))$

p.306 l.-12: replace $\{\pi_{s,x}(\kappa_n) \colon n < \omega, s \subset x, \mathrm{lh}(s) = n+1\}$ by $\{\pi_{s,x}(\kappa_n) \colon n < \omega, s \subset x, \mathrm{lh}(s) \geq n+1\}$. (Thanks to Andreas Lietz!)

p.312 l.1f: add $s' \subset s''$ and $t' \subset t''$ as hypotheses.

p.312 displayed formula in the statement of (PD, 4): there is a ")" missing and to be put at the very end.

p.317 last line of (13.18): $\pi_{(\emptyset,\emptyset),(s\restriction k,t)}$ needs to be replaced by $\sigma_{(\emptyset,\emptyset),(s\restriction k,t)}$.

p.317 second line of (13.20): $\pi_{2i\dot{-}1,2m\dot{-}1}$ needs to be replaced by $\pi 2i\dot{-}1, 2n+1$.

p.319 l.6: $V_\lambda^{M_{2n-1}}$ needs to be replaced by $V_\lambda^{M_{2n+1}}$

p.319 l.3 from bottom: the λ^* here is not the λ^* from the middle of p.317.

p.321 2nd line of the displayed formula in (H): $\pi_{(s\restriction i,t\restriction i),(s\restriction k,t)}$ needs to be replaced by $\sigma_{(s\restriction i,t\restriction i),(s\restriction k,t)}$. (Thanks to Andreas Lietz for the last 7 typos!)