Errata of the book
“Set theory. Exploring Independence and Truth”

by Ralf Schindler

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p.2 l.-6: delete the first “u” in “analoguous.”

p.4, l.8f.: the definition should read: “A set $B \subset A$ is called *dense in* A iff for all a, $b \in A$ with $a < b$ and $(a, b) \cap A \neq \emptyset$, then $(a, b) \cap B \neq \emptyset$. (Thanks to Milad Khodayi!)

p.5, line following the statement of Corollary 1.10: should read “Proof of Theorem 1.9,” not “Proof of Theorem 1.8.”

p.5 l.16: delete the last “that.”

paragraph at the bootom of p.6 and the top of p.7: delete the sentence “As $\mathbb{Q}$ is dense [...] picked to be pairwise disjoint.”

p.8 l.-2: delete “$[a, b]_\infty$ is dense in $[a, b]$.” This is obvious nonsense. (Thanks to Alexander Paseau!)

p.18 l.4: Suppose that b does *not* have a maximum [...].

p.18 l.-7: delete “the.”

p.20 l.10: Shat *that* [...]

p.23 l.8: insert “is” before “inductive.”

p.27 l.9: replace “the R–least x_0” by “an R–least x_0.” (Thanks to Philipp Schlicht!)

p.34 l.1: “my” should be “may.”

p.35 l.3f.: ... for cardinals κ, λ *with* $\lambda \leq \kappa$.

p.35 l.20: replace $\pi(\gamma)$ by $\pi((\gamma, \gamma))$. Similarly, l.25: replace $\pi(\aleph_0)$ by $\pi((\aleph_0, \aleph_0))$, l.27: relace $\pi(\aleph_\alpha)$ by $\pi((\aleph_\alpha, \aleph_\alpha))$.

p.37 l.16: replace “is” by “in.”

p.38 l.7: replace “Poblem” by “Problem.”

p.41 l.-3: replace $A_{x\eta_n}$ by A_{η_n}.

p.43 l.1: replace “from” by ”form.”

p.43 l.2: replace κ^+ by κ.

p.44 l.3: replace $\gamma_\alpha^{\alpha'} \bar{S}$ by $\gamma_\alpha^{\alpha'} \in \bar{S}$.

p.44 l.-10: replace $g_i \colon [\mu_i]^{\mathrm{cf}(\kappa)} \to \mu_i^+$ by $g_i \colon [\mu_i]^{\leq \mathrm{cf}(\kappa)} \to \mu_i^+$.

p.44 l.-3: unfortunately, this is not the same g_i as in l.-10 of the same page.

p.45 l.2: delete "in."

p.45 l.11 and l.15: replace $\mathcal{P}(\kappa)$ by $[\kappa]^{\mathrm{cf}(\kappa)}$; also lines 11, 14, 15, and 16: replace $Y \subset \kappa$ by $Y \in [\kappa]^{\mathrm{cf}(\kappa)}$.

p.49f.: The construction of a κ^+ Aronszajn tree is imprecise. Let us fix it as follows.

Let $(A_s \colon s \in {}^{<\omega}\kappa)$ be such that $A_\emptyset = \kappa$ and for all $s \in {}^{<\omega}\kappa$, $\{A_{s^\frown\xi} \colon \xi < \kappa\}$ is a family of pairwise disjoint sets with $A_s = \bigcup\{A_{s^\frown\xi} \colon \xi < \kappa\}$. Let

$$\mathcal{A} = \{\cup B \colon B \in [\{A_s \colon s \in {}^{<\omega}\kappa \wedge s \neq \emptyset\}]^{<\kappa}\}.$$

By $\kappa^{<\kappa} = \kappa$, $\mathrm{Card}(\mathcal{A}) = \kappa$.

Now replace items (1), (2), and (4) on p.49 by the following.

(1) For all $s \in T$ there is some $A \in \mathcal{A}$ with $\mathrm{ran}(s) \subset A$.

(2) If $s \in T$, $\mathrm{ran}(s) \subset A \in \mathcal{A}$, $B \in \mathcal{A}$, $B \cap A = \emptyset$, $\mathrm{lv}_T(s) < \beta < \kappa^+$, then there is some $t \in T$ with $\mathrm{lv}_T(t) = \beta$, $s \subset t$, and $\mathrm{ran}(t) \subset \mathrm{ran}(s) \cup B$.

(4) Let $\lambda < \kappa^+$ be a limit ordinal with $\mathrm{cf}(\lambda) < \kappa$. Let $C \subset \lambda$ be club in λ with $\mathrm{otp}(C) = \mathrm{cf}(\lambda)$, and let $(\lambda_i \colon i < \mathrm{cf}(\lambda))$ be the monotone enumeration of $\{0\} \cup C$. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be a pairwise disjoint family of elements of $\mathcal{A}$. Let $s \colon \lambda \to \kappa$ be such that

$$s \restriction \lambda_i \in T_{\lambda_i+1} \wedge s\text{''}\lambda_i \subset \bigcup\{A_j \colon j < i\}$$

for every $i < \mathrm{cf}(\lambda)$. Then $s \in T_{\lambda+1}$.

The rest is as before except that in case $\mathrm{cf}(\lambda) = \kappa$ we pick $s(t)$ as follows. We fix $C \subset \lambda$, a club in λ with $\mathrm{otp}(C) = \kappa$, and we let $(\lambda_i \colon i < \kappa)$ be the monotone enumeration of $\{0\} \cup C$. By the new (1), $\mathrm{ran}(s) \subset A \in \mathcal{A}$ for some A. Let $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{B\}$ be such that $\{A_i \colon i < \mathrm{cf}(\lambda)\} \cup \{A, B\}$ is a pairwise disjoint family of elements of $\mathcal{A}$. (This choice is possible!) Using the new (2) and the new (4), we may construct some $t \colon \lambda \to \kappa$ extending s such that for every $i < \kappa$,

$$t \restriction \lambda_i \in T_{\lambda_i+1} \wedge t\text{''}\lambda_i \subset A \cup \bigcup\{A_j \colon j < i\}.$$

We write $t(s)$ for this t. We then let $T_{\lambda+1} = T_\lambda \cup \{t(s) \colon s \in T_\lambda\}$.

p.54 l.8: $\{\xi < \kappa \colon f(\xi) \in g(\xi)\} \in U$

p.56 l.4: Th*e*orem

p.62, Problem 4.4: cf. p.35 l.3f.

p.64, Problem 4.25: This is wrong for trivial reasons. The statement has to be adjusted as follows. If κ is ineffable, then there is no slim κ-Kurepa tree. Here, a κ-tree T is *slim* iff for all $\alpha < \kappa$, $T_\alpha = \{s \in T \colon \mathrm{lv}_T(s) < \alpha\}$ has size at most $\mathrm{Card}(\alpha)$. (Thanks to Shervin Sorouri!)

p.97 footnote 1: replace "until p. 97" by "until p. 101."

pp.72f. and p.88: The proof of Lemma 5.11 is wrong, as not every rud_E function is simple in the sense of the definition of "simple" given in "$\subset$" of that proof. E.g.

if $f(\vec{x})$ is rud_E, then the formula $f(\vec{x}) \in E$ need not be Σ_0 in the language $\mathcal{L}_{\in,E}$. (Thanks to Shervin Sorouri!)

This proof should be fixed as follows. First,

$$\mathcal{P}(U) \cap \underset{\sim\omega}{\Sigma}(U; \in, E) = \mathcal{P}(U) \cap \underset{\sim 0}{\Sigma}(U \cup \{U, E\}; \in),$$

so that we have to prove that

$$\mathcal{P}(U) \cap \mathrm{rud}_E(U \cup \{U\}) = \mathcal{P}(U) \cap \underset{\sim 0}{\Sigma}(U \cup \{U, E\}; \in).$$

In the proof of "$\subset$," the definition of "simple" then has to be adjusted as follows: Let a formula φ in $\mathcal{L}_{\in,E}$ be Σ_0' iff φ is in the smallest class Γ of formulas such that

(a) all atomic formulas are in Γ,

(b) Γ is closed under sentential connectives,

(c) Γ is closed under bounded quantification, and

(d) if ψ is in Γ and x is a variable, then $\exists x \in E\, \psi$ and $\forall x \in E\, \psi$ are both in Γ.

These are the closure conditions as in Definition 5.1 on p.67 *plus* that quantification over elements of E counts as bounded quantification.

Now call a function $f \colon V^k \to V$, where $k < \omega$, *simple* iff the following holds true: if $\varphi(v_0, v_1, \ldots, v_m)$ is Σ_0' in the language $\mathcal{L}_{\in,E}$, then $\varphi(f(v_1', \ldots, v_k'), v_1, \ldots, v_m)$ is equivalent over rud_E closed structures to a Σ_0' formula in the same language.

This is also the definition of "simple" which should be used in Problem 5.8 on p.88.

p.106 l.19: add ", and $\{\xi^p_{k_0} : p \in D_1\}$ is unbounded in ω_1." (Let $\beta < \omega_1$ be such that $\xi^p_k < \beta$ for all $p \in D_0$ and $1 \leq k < k_0$. If for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and for all $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$ the set $\{\xi^p_{k_0} : p \in D_0 \wedge \xi^p_1 = \xi_1 \wedge \ldots \wedge \xi^p_{k_0-1} = \xi_{k_0-1} \wedge p(\xi_1) = s_1 \wedge \ldots \wedge p(\xi_{k_0-1}) = s_{k_0-1}\}$ were bounded in ω_1, then there would be one common bound for all $\xi_1 < \ldots \xi_{k_0-1} < \beta$ and $s_1, \ldots, s_{k_0-1} \in {}^{<\omega}\omega$, contradicting the choice of k_0.) In l.23f., replace "By the choice of k_0" with "By the choice of D_1."

p.118 Corollary 6.62: Add "$\mathbb{P}$ is separative" to the hypotheses in the statement of this corollary, as its proof makes use of Lemma 6.60. (Thanks to Fan Feng!)

p.120 l.5 from b.: replace "Lemma 6.65" by "Lemma 6.32." (Thanks to Fan Feng!)

p.137f., proof of Claim 7.18: The construction of U is wrong. Let $\delta, \epsilon \mapsto \langle \delta, \epsilon \rangle$ be the GÖDEL pairing function, see p.35. Then let $(s, t) \in U$ iff for all $n < \omega$, $(s \upharpoonright k, n, (t(\langle n, 0 \rangle), \ldots, t(\langle n, k-1 \rangle))) \in U$, where $k \geq 0$ is maximal such that $k = 0$ or $\langle n, k-1 \rangle < \mathrm{lh}(t) = \mathrm{lh}(s)$. (Thanks to Andreas Lietz and Stefan Hoffelner!)

p.139 l.7 from b.: This should say "Also, if $(2^{\aleph_0})^{L[x]} = \omega_1^{L[x]} < 2^{\aleph_0}$, then by Lemmas 7.19 and 7.20 there is a *largest* $\Sigma^1_2(x)$–set of reals which is smaller than $2^{\aleph_0}$, namely ${}^\omega\omega \cap L[x]$."

p.140: the 2nd last displayed formula on that page got screwd up. The aim is to choose x such that $(\varphi_0(x), x(0), \varphi_1(x), x(1), \dots)$ is $<_{\text{lex}}$–minimal. Let the formula read:

$$[x \upharpoonright n = y \upharpoonright n \wedge \forall m < n(\varphi_m(x) = \varphi_m(y))] \longrightarrow$$

$$[y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y) \vee (\varphi_n(x) = \varphi(y) \wedge x(n) \leq y(n)))].$$

As explained in the text, for $x \in A$, "$y \notin A \vee (y \in A \wedge (\varphi_n(x) < \varphi_n(y)))$" and "$y \notin A \vee (y \in A \wedge (\varphi_n(x) \leq \varphi_n(y)))$" can both be uniformly written in a Π^1_1 as well as in a Σ^1_1 way, so that the relevant formula is Π^1_1. (Thanks to Robin Puchalla!)

p.162 last two lines: The proof of $N \models \mathsf{DC}$ has to be fixed as follows. Let R, $a \in N$ be such that for all $x \in a$ there is a $y \in a$ with $(x, y) \in R$. Let us work in $V[G]$ and define $f\colon \omega \to a$ as follows. For each $n < \omega$,

$$f(n) = \{u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi_n(u, \beta_n, x_n)\},$$

where $(\ulcorner\varphi_n\urcorner, \alpha_n, \beta_n)$ is the $<_{\text{lex}}$-least $(\ulcorner\varphi\urcorner, \alpha, \beta)$ such that there is some $y \in {}^\omega\omega$ such that

(a) $\{u \in V_\alpha \colon V_\alpha \models \varphi(u, \beta, y)\} \in a$, and

(b) if $n > 0$, then $(x_{n-1}, \{u \in V_\alpha \colon V_\alpha \models \varphi(u, \beta, y)\}) \in R$

and $x_n \in {}^\omega\omega$ is such that

(a) $\{u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n)\} \in a$, and

(b) if $n > 0$, then $(x_{n-1}, \{u \in V_{\alpha_n} : V_{\alpha_n} \models \varphi(u, \beta_n, x_n)\}) \in R$.

Let $x = \oplus_{n<\omega} x_n$. It is then easy to see that f is definable in $V[G]$ from x together with an ordinal and a real parameter which defines (a, R). Hence $f \in N$.

p.170 l.14: replace $C \cap U_s \cap \mathcal{O}^x_n = \emptyset$ by $C \cap U_s \cap \mathcal{O}^x_n \neq \emptyset$. (Thanks to Vincenzo Dimonte!)

p.180 l.2 from b.: replace $g(n) \leq m < g(n+1)$ by $f(n) \leq m < f(n+1)$

p.189 l.10: replace $[Y]^{<\omega}$ by $[Y]^n$

p.193: the 2nd displayed formula should read $h\colon \bigcup_{n<\omega}(\{n\} \times {}^{v(n)}M) \to M$

p.195 l.6 from b.: replace $\{\xi < \kappa \colon f(x) \in y\}$ by $\{\xi < \kappa \colon f(\xi) \in y\}$. (Thanks to Vincenzo Dimonte!)

p.199 l.7 from b.: replace "Lemma 1.31 (g) and (e)" by "Lemma 10.21 (g) and (e)." (Thanks to Vincenzo Dimonte!)

p.199 l.2 from b.: replace "Lemma 10.29 (h)" by "Lemma 10.21 (h)." (Thanks to Vincenzo Dimonte!)

p.203 l.3 from b.: replace "In (c)" by "In (iii)" (Thanks to Vincenzo Dimonte!)

p.210 Definition 10.45: It has to be added that if E is a (κ, ν)–extender, then ν is called the *length of* E. The concept of the length of an extender gets used e.g. in the proof of Theorem 10.74. (Thanks to Bob Lubarsky!)

p.212 l.5f.: $g\colon [\mu_b]^{\mathrm{Card}(b)} \to M$

p.226: U^* refers to two different things on this page, to a tree, defined l.9, and to a substructure of R_i, defined l.17 (display). Also, τ refers to two different things on this page, to $\sigma' \restriction V_{\nu_i}^{\bar{M}_i}$, defined l.9, and to a map from (the 2nd) U^* to $V_\kappa^{R_i}$, defined l.18. There is also a sloppyness about Σ_{1+} formulae on this page in that the first parameter (free variable) of Φ got suppressed: e.g. in (10.46) by $\Phi(\sigma_i \restriction V_{\nu_i}^{\bar{M}_i})$ I really meant $\Phi(\sigma_i(\nu_i), \sigma_i \restriction V_{\nu_i}^{\bar{M}_i})$, i.e., $\Phi(\tau)$ in l.7 should have been written as $\Phi(\tau(\nu_i), \tau)$ – with the understanding that $\tau(\nu_i) = \sup(\tau"\nu_i)$. (Thanks to Bob Lubarsky!)

p.239 Lemma 11.13: Make "$\forall x \in U' \, \exists y \in U' \, x \in y$" part of the hypothesis. Without this additional hypothesis (a) and (c) are false: Take $U = 4$, $U' = 4 \cup \{\{0,2\}\}$, and $\pi = \mathrm{id}$. (Thanks to Toby Meadows!)

p.240 1st displayed formula in the proof of Lemma 11.16: $\pi(\xi)$ should be $\pi(\mathbf{z})$.

p.241 l.10 from bottom: $[\rho_1(\bar{M})]^{\geq\omega}$ should be $[\rho_1(\bar{M})]^{<\omega}$.

p.245 middle: in the definition of $[n, x]$, $[m, y]$ needs to be replaced by (m, y).

p.246 upper half middle: in the definition of $[n, x]^*$, $[m, y]$ needs to be replaced by (m, y).

p.249 l.9 from bottom: delete "$r\bar{l}$" at the beginning of the displayed formula.

p.250 l.2 from bottom: in the definition of Γ_M^{n+1}, the superscript $\geq \omega$ needs to be replaced by $< \omega$.

p.252 l.11 from bottom (displayed formula): $h_{M''}^{n+1,p}(\omega \times {}^{<\omega}X)$ needs to be replaced by $h_M^{n+1,p}\,"(\omega \times {}^{<\omega}X)$.

p.255 l.17: replace $q(n)$ by $q(n-1)$.

p.257 proof of Lemma 11.43: There should be a case split in this proof, as the proof which is written down implicitly assumes that $\nu \in W_M^{\nu,p}$. If $\nu = \max(p)$, then it might be possible that $\nu = W_M^{\nu,p} \cap \mathrm{OR}$, in which case $W_M^{\nu,p} = M|\nu \in M$ is trivial, though. (Thanks to Andreas Lietz!)

p.257 l.5-4 from bottom: "$\sigma(\nu)$ is regular in M" needs to be replaced by "$\tau(\nu)$ is regular in M." Also, l.2-1 from bottom: all three occurances of $J_{\sigma(\nu)}[B]$ should be replaced by $J_{\tau(\nu)}[E']$.

p.275 Problem 11.3: Cf. the correction to p.239 Lemma 11.13.

p.280 l.8: $z \in {}^\omega X$

p.280: the displayed formula in the middle should read $\{z * \tau \colon z \in {}^\omega X\} \subset {}^\omega\omega \setminus A$

p.282 l.1: $s = (x_0, \dots, x_{n-1})$

p.306 l.-12: replace $\{\pi_{s,x}(\kappa_n) \colon n < \omega, s \subset x, \mathrm{lh}(s) = n+1\}$ by $\{\pi_{s,x}(\kappa_n) \colon n < \omega, s \subset x, \mathrm{lh}(s) \geq n+1\}$. (Thanks to Andreas Lietz!)

p.312 l.1f: add $s' \subset s''$ and $t' \subset t''$ as hypotheses.

p.312 displayed formula in the statement of (PD, 4): there is a ")" missing and to be put at the very end.

p.317 last line of (13.18): $\pi_{(\emptyset,\emptyset),(s\restriction k,t)}$ needs to be replaced by $\sigma_{(\emptyset,\emptyset),(s\restriction k,t)}$.

p.317 second line of (13.20): $\pi_{2i\dot{-}1,2m\dot{-}1}$ needs to be replaced by $\pi 2i\dot{-}1, 2n+1$.

p.319 l.6: $V_\lambda^{M_{2n-1}}$ needs to be replaced by $V_\lambda^{M_{2n+1}}$

p.319 l.3 from bottom: the λ^* here is not the λ^* from the middle of p.317.

p.321 2nd line of the displayed formula in (H): $\pi_{(s\restriction i,t\restriction i),(s\restriction k,t)}$ needs to be replaced by $\sigma_{(s\restriction i,t\restriction i),(s\restriction k,t)}$. (Thanks to Andreas Lietz for the last 7 typos!)