Correction: the argument for Claim 16 on p. 55 f. is bogus. The argument is supposed to use the forcing theorem to get a \( p \in \mathcal{P} \) with

\[
\mathbb{P}_{\mathcal{P}} \vdash \forall \exists \mathcal{P} (\exists \mathcal{P} \leftrightarrow \mathcal{P}^{\mathcal{P}} (\exists, \mathcal{P})) ,
\]

ct. p. 56 lines 1-2. To prove the relevant instance of the forcing theorem, though, we'd need dense sets to exist in \( \mathcal{P}^{\mathcal{P}} \) which are definable over \( \mathcal{P}^{\mathcal{P}} \) by formulae whose complexity is at least the complexity of \( \mathcal{P} \), but \( \mathcal{P} \) witnesses \( \mathcal{P}^{\mathcal{P}} (\mathcal{P}^{\mathcal{P}}) \leq \mathcal{P} < \mathcal{P}^{\mathcal{P}} \). The argument is thus circular at best. The revised \( P \)-construction which is proposed on pp. 59 ff. is suspicious for other reasons as well.

The proof of Claim 21 is not affected by this, though.
We now discard pp. 55–59 and go back to the task of reorganizing \( \nu_0 = \mathcal{L}[\mu^0_\infty, P \mapsto P] \),
\[ = \text{HOD}^{\text{Con}}(\omega, < \kappa_0) \]

We know that \( M \) is \( P \)-generic on \( \nu_0 \) for some \( P \in \nu_0 \), \( P \subseteq \nu_0 \upharpoonright \kappa_0^{++} \), \( \nu_0 \models "P \text{ has the } \kappa_0^+ M \text{-c.c.}" \) (and \( \kappa_0^+ M = \delta_0 \mu^0_\infty \), \( \kappa_0^{++} M = \kappa_0^{++} \mu^0_\infty \)).

Let us do a revised \( P \)-construction above \( \nu_0 \upharpoonright \kappa_0^{++} \), inside \( M \), as follows. Let \( O \upharpoonright \nu \),
\( O \upharpoonright \nu \) denote the passive, active levels of this \( P \)-construction, respectively.

While this construction is performed inside \( M \),
we aim to inductively maintain that the following are satisfied.

\((*)\) \(P \mid \nu, P \parallel \nu \in \mathcal{L}_0\) for all \(\nu\),

\((**)\) \(P \mid \nu [g] = M \mid \nu, P \parallel \nu [g] = M \parallel \nu\)

for all \(\nu\), while \(g = M \check{x}_{\omega+2}\) is \(P\)-generic on \(P \mid \nu\) (\(P\) as above).

Notice that by \((*)\), \(P \mid x_{\omega+3}^+ = \nu \mid x_{\omega+3}^+\), which automatically gives that \(g\) is \(P\)-generic on \(P \mid \nu\).

(Claim 16 on p. 55 will thus be true for the new \(P\); \((**)\) is a restatement of Claim 17; Claim 18 is then given by Hamkins-Laver-Woodin, cf. Lemma 2 of the appendix.)

To describe our new revised \(P\)-construction, we only have to say how to get from \(P \mid \nu\) to \(P \parallel \nu\).

Thus, let \(\nu \geq x_{\omega+3}^+\), and let \(P \mid \nu\) be given such that \(P \mid \nu \in \mathcal{L}_0\) and \(P \parallel \nu [g] = M \mid \nu\).
If \( \nu \) indexes an extend with critical point \( > \kappa_0^{+3} \) (which is equivalent to the fact that \( E^M_\nu \) has critical point \( \kappa_0 \)), then we let \( E^M_\nu \cap P|\nu \) be the top predicate of \( P|\nu \).

In order to get (\( *) \), we need that

(A) the extension of \( E^M_\nu \) to \( M^{Con(\omega, < \kappa_0)} \) is OD in \( M^{Con(\omega, < \kappa_0)} \).

This is true because if (A) holds, then by

\( P|\nu \in \mathcal{L}_0 = \text{HOD}^{M^{Con(\omega, < \kappa_0)}} \), \( E^M_\nu \cap P|\nu = (\text{the extension of } E^M_\nu \text{ to } M^{Con(\omega, < \kappa_0)}) \cap P|\nu \in \text{HOD}^{M^{Con(\omega, < \kappa_0)} \mathcal{L}_0} = \mathcal{L}_0 \).

On the other hand, suppose that \( E^M_\nu \cap P|\nu \in \mathcal{L}_0 \).

Let \( a \in [\kappa]^{<\omega} \), and let \( X \) be in the \( a \)-th component of the extension of \( E^M_\nu \) to \( M^{Con(\omega, < \kappa_0)} \), \( X \in M^{Con(\omega, < \kappa_0)} \), say \( X = \tau^{g \times h} \), where \( h \) is \( Con(\omega, < \kappa_0) \)-generic over \( M = \mathcal{L}_0[G] \). By closure, there is one \( p \in g \times h \) s.t. \( \{ u \in [\text{crit}(E^M_\nu)]^a : p \forces_{\nu_0} \text{P} \} \in g \times h \). 

\( \in E^M \). In other words, there is some \( \overline{X} \in (E^M \cap \mathcal{P} \nu)_a \), \( \overline{X} \in \mathcal{P} \nu \), which "forces" \( X \) to be in the \( a \)th component of the extension of \( E^M \) to \( M^{\mathcal{C} \nu}(w, \omega_0) \) in the sense that \( \overline{X} \subseteq X \), and hence this extension is definable in \( M^{\mathcal{C} \nu}(w, \omega_0) \) from the parameter \( E^M \cap \mathcal{P} \nu \). Hence if \( E^M \cap \mathcal{P} \nu \in \nu = \text{HoDM}^{\mathcal{C} \nu}(w, \omega_0) \), then (A) holds true. (A) is thus equivalent to \( E^M \cap \mathcal{P} \nu \in \nu \).

We defer the proof of (A).

Let us now suppose that \( \nu \) indexes an extende with critical point \( = \kappa_0 \). We then let the top predicate of \( \mathcal{P} \nu \) code

\[
\left\{ \left( \mathcal{I}, \Sigma^{\mu_0 \cdot \omega_0}(\mathcal{I}) \right) : \mathcal{I} \in \mathcal{P} \nu \right\} \text{ a tree of limit length on } \omega_0 \cdot \omega_0 \cdot \omega_0.
\]
in an amenable way (this is possible as
\( c(\nu) = c(\kappa^+_0) = \delta^{\omega_0}_0 \text{ in } M \)). Here, \( \kappa^+_0 \) is
the largest cardinal of \( \sigma^\nu \).

By Subclaim 2 on p. 53, we certainly have \((*)\).

Let us argue that \( P^{\|\nu}[\vec{g}] = M^{\|\nu} \). We
have to see that \( E^M_\nu \) is definable on \( P^{\|\nu}[\vec{g}] \).

Let \( \vec{t} \in M^{\|\nu} \) be a tree of limit length on
\( \omega_0 \setminus \delta^{\omega_0}_0 \), say \( \vec{t} = t^\bar{\alpha} \), \( t^\bar{\alpha} \in (P^{\|\nu})^P \). Let \( U \)
on \( \omega_0 \setminus \delta^{\omega_0}_0 \) arise from composing all \( M(t^\bar{\alpha}) \)
with \( \omega_0 \setminus \delta^{\omega_0}_0 \), where \( g^\kappa \) is the finite variant of
\( g \) given by \( s \). Then \( U \in P^{\|\nu} \), and hence
\( b = \Sigma_{\omega_0 \setminus \delta^{\omega_0}_0}(U) \) is given by the \( ^+P \) predicate of
\( P^{\|\nu} \). But then \( c = \Sigma_{\omega_0 \setminus \delta^{\omega_0}_0}(\vec{t}) \) is the unique
branch \( c \) thru \( \vec{t} \) s.t. there is an embedding
\( k: \mathcal{U}_c^\vec{t} \rightarrow \mathcal{U}_b^\vec{t} \) with \( k \circ \pi_{\vec{t}}^{\mathcal{U}_c} = \pi_{\vec{t}}^{\mathcal{U}_b} \).
(Cf. Lemma 2.1 of [\& VMJ].)
Let $\alpha$ be the next admissible after $\nu$, i.e., $M|\alpha = KP$. Then $\mathcal{P}|\alpha \eta$ can identify a direct limit system whose direct limit is exactly

$$
\pi_{E^M_\nu} (\mu_0^\infty | \mu_0^\infty)$. But then

$$
\pi_{E^M_\nu} (\mu_0^\infty | \mu_0^\infty) \colon \mu_0^\infty | \mu_0^\infty \rightarrow \pi_{E^M_\nu} (\mu_0^\infty | \mu_0^\infty)
$$

is in $\mathcal{P}|\alpha + \omega$. Let $e : \kappa_0^+ \leftrightarrow M|\kappa_0^+$ be the canonical enumeration which is given by the can. w.o. of $M|\kappa_0^+$, so that $e$ is $\Sigma_1^M M|\kappa_0^+$. Then

$$
\pi_{E^M_\nu} (e) : \nu \leftrightarrow M|\nu$ is the can. enumeration given by the can. w.o. of $M|\nu$. $Y = \pi_{E^M_\nu} (X)$ iff there is some $\xi < \kappa_0^+$ s.t. $X = e(\xi)$ and $Y = \pi_{E^M_\nu} (e) (\pi_{E^M_\nu} (\xi))$.

As $e$, $\pi_{E^M_\nu} (e)$, and $\pi_{E^M_\nu} | \kappa_0^+$ are in $\mathcal{P}|\alpha \omega_\nu [g]$, $E^M_\nu$ is in $\mathcal{P}|\alpha [g]$ also, i.e., $\mathcal{P}|\alpha_\nu [g] = M|\alpha + \omega$.

This basically shows (***) if we reinterpret $\mathcal{P}|\alpha [g] = M|\nu$, as meaning that $\mathcal{P}|\alpha + \omega [g] = M|\alpha + \omega$. 
If $E_k^0 = \emptyset$, we simply construct one step further.

Modulo (A) on p. 74, we thus showed how to reorganize $\kappa_0$; (A) was shown to hold true by Farmer Schützenberg by generalizing arguments from his Ph.D. thesis.

The revised $\beta$-construction is presented on pp. 72–78 is a construction above $\kappa_0 + 3$. It does not gain any information as to how $\kappa_0$ could be organized as some kind of useful "premouse" below $\kappa_0 + 3$. We're now going to do exactly that.

Recall that $\kappa_0 = L[\kappa_0^0, \kappa_0^1, \kappa_0^2, \kappa_0^3]$, and $\rho \mapsto \rho^*$ may be computed from $\kappa_0^0$ and $(\rho \mapsto \rho^*)|_{\kappa_0^0}$. $\rho \mapsto \rho^*$ is the map from $\kappa_0^0$ into its own $\kappa_0^0$ (restricted to the ordinals).
Let $F$ be the least $M$-measure on $\kappa_0$.

Let $\mu^0_\infty = (\mu^0_0)^{\mu^0_0}$, and let $\kappa_0^+ + \mu^0_0$ be the $\mu^0_0$-successor of the least strong of $\mu^0_0$, so that $(\rho \mapsto \rho^+) \upharpoonright \delta_0^{\mu^0_0} : \delta_0^{\mu^0_0} = \kappa_0^+ \rightarrow \kappa_0^+ + \mu^0_0$ is cofinal.

Claim 22. $\kappa_0^+ + \mu^0_0 < \kappa_0^{++} \cdot \text{ult}(M; F)$.

Proof: Let $\alpha$ be least s.t. $\mathcal{I}_{\alpha} [\mu^0_0, \kappa_0^+ + \mu^0_0, \rho \mapsto \rho^+ \upharpoonright \delta_0^{\mu^0_0}]$ is a $2FC_\alpha$-model. We show that $\alpha < \kappa_0^{++} \cdot \text{ult}(M; F)$.

$\text{ult}(M; F)$ can compute $\mu^0_0 \upharpoonright \delta_0^{\mu^0_0}$ as well as $(\rho \mapsto \rho^+) \upharpoonright \delta_0^{\mu^0_0}$. By the argument on pp. 9-12,
\[ J_\delta \left[ \omega_1^\omega \mid \kappa^+ \omega_\delta, \rho \vdash \rho^+ \left[ \delta_0 \omega_\delta \right] \right] \text{ knows that every ordinal } \in \left[ \kappa^+, \kappa^+ \omega_\delta \right] \text{ has size } \kappa^+_0 = \delta_0 \omega_\delta, \]

and of course it knows that \( \kappa^+ \omega_\delta \) has cardinality \( \kappa^+_0 = \delta_0 \omega_\delta \).

Suppose that \( \kappa^+ \omega_\delta \) were a cardinal in
\[ J_\alpha \left[ - \right] = J_\alpha \left[ \omega_1^\omega \mid \kappa^+ \omega_\delta, \rho \vdash \rho^+ \left[ \delta_0 \omega_\delta \right] \right], \]
i.e. \( \kappa^+ \omega_\delta = \left( \omega_0^\omega + \omega \right) \). Let \( I \) on \( \omega_0^\omega / \delta_0 \omega_\delta \) be the canonical tree of length \( \kappa^+ \omega_\delta \) to make \( \omega_0^\omega / \kappa^+ \omega_\delta \)
geometric, so that \( I \) is definable over \( \omega_0^\omega / \kappa^+ \omega_\delta \).

Let \( b = \Sigma_{\omega_0^\omega / \delta_0 \omega_\delta} (I) \).

I claim that \( b \in J_\alpha \left[ - \right] \). As \( \kappa^+ \omega_\delta = \left( \omega_0^\omega + \omega \right) \),
we may take, inside \( J_\alpha \left[ - \right] \),
\[ N \models \sigma \prec X \prec J_\alpha \left[ - \right] \]
\( \sigma \prec X \prec \tilde{\delta} \)
s.t. \( X \cap \kappa^+ \omega_\delta \in \kappa^+ \omega_\delta \), \( I \in X \), etc., and \( N \) is transitive. Let \( \sigma(I) = \tilde{I} \).
\[ J \{ - \} \] can see \( \Sigma_{\omega_1} \Sigma_{\omega_1} \) (call it \( c \)), by searching for \( c' \) s.t. there is \( \omega_1 \rightarrow \omega_1^\omega \). 

As we may assume \( \mathcal{X} \cap \mathcal{R} \) is cofinal 
in \( \kappa_0^+ \omega_0^+ \) (recall \( \mu(\kappa_0^+ \omega_0^+) = \delta_0 \omega_0^+ < \kappa_0^+ \omega_0^+ \) in \( J \{ - \} \), \( \omega_2 \leq c' \in J \{ - \} \) induces a cofinal branch thru \( J \). By branch condensation, this branch is equal to \( b \).

But now the argument pp. 9-12 again gives that \( J \{ - \} \) knows that \( \text{Card}(\kappa_0^+ \omega_0^+) = \kappa_0^+ \).

As everything can be done in \( \text{ult}(M; F) \), \( \kappa_0^+ < \omega_2 + \text{ult}(M; F) \) as desired. \( \sim \) (claim 22)

In particular, \( \forall F \in \omega_0 \Sigma_{\omega_1} \) (\( \rho \rightarrow \rho^* \)) \( \Sigma_{\omega_1} \). 

Let's still write \( \alpha \) for the least \( \alpha \) s.t. \( J \{ \omega_0 \} \kappa_0^+ \omega_0^+, (\rho \rightarrow \rho^* ) \| \delta_0 \omega_0^+ \) is a 2FC-model. 

So \( \alpha < \kappa_0^+ \omega_2 + \text{ult}(M; F) \), and hence \( \alpha \) is smaller than the order of \( F \) (no matter if we use
Jensen or Mitchell-Steel indexing.

Claim 23. Let's write \( Q = \mathcal{J}_{\omega_1 + \omega_2}^{\omega_1 + \omega_2}(\mathcal{P}(\mathfrak{p}^*))^{\omega_1 + \omega_2} \).

\( \mathbb{M}^{\alpha} \) is a generic extension of \( Q \).

Proof: It is not hard to show that \( Q \) is a definable inner model of \( \mathbb{M}^{\alpha} \). We claim that for each function \( f : \theta \rightarrow \text{OR} \), \( f \in \mathbb{M}^{\alpha} \) (cho arbitrary), there is some \( g \in Q \), \( \text{dom}(g) = \theta \), \( f(\xi) \in g(\xi) \)

and \( g(\xi) \leq \omega_2 \) for all \( \xi < \theta \). Otherwise let \( f \) be the least countable, so that \( f(\xi) = \gamma \) if

\( \mathbb{M}^{\alpha} \models \gamma(\xi, \gamma) \). But then we may let

\( g(\xi) = \{ \gamma : \exists \phi \text{ ext. alg. } \mathcal{M}^{\omega_1 \alpha} \models \gamma(\xi, \gamma) \text{ definable, and } \gamma(\xi^*, \gamma^*) \} \).

This gives a function \( g \) as desired and a contradiction.

The result now follows by Bukovsky.
According to Schützenberg, \( \kappa_0^{+} m_0^0 \) is the least \( \alpha \) s.t. \( J_\alpha [M | \kappa_0^{+}] \) is admissible.

We now perform a revised P-construction above

\[
J_\alpha [ m_0^{0} | \kappa_0^{+} m_0^0, (\rho \rightarrow \rho^*) | \delta_0 m_0^0 ]
\]

in much the same way as on pp. 72—78.

We set \( \mathcal{P} | \alpha = J_\alpha [ m_0^{0} | \kappa_0^{+} m_0^0, (\rho \rightarrow \rho^*) | \delta_0 m_0^0 ] \).

Having constructed \( \mathcal{P} | \alpha \), we proceed as follows.
1st case. $E^M_\omega = \varnothing$.

Then we just construct one step further.

2nd case. $E^M_\omega \neq \varnothing$ has critical point $\neq \kappa_0$.

Then $\text{crit}(E^M_\omega) > \omega$, and we let

$$P_{II\nu} = (P_{II\nu} ; E^M_\omega \cap P_{II\nu})$$

3rd case. $E^M_\omega \neq \varnothing$ has critical point $= \kappa_0$.

Then we let

$$P_{II\nu} = (P_{II\nu} ; S^P_\omega)$$

where $S^P_\omega = \{ (\overrightarrow{t}, \Sigma_{\omega_0} \cup_{\omega_0} (\overrightarrow{t}) ) : \overrightarrow{t} \in P_{II\nu}^P \}$,

$a$ tree of limit length on $\omega_0 \cup_{\omega_0} \omega_0$,

$\lambda_{P_{II\nu}}$ the largest cardinal of $P_{II\nu}$.

We inductively maintain that all $P_{II\nu}$ are in $\text{HOD}^{M^{\text{cn}(\omega,<\kappa_0)}} = V$ and that

$M_{II\nu} = P_{II\nu}[g]$ for some $g$ which is $P_{II\nu}^P$ generic.

on $P_{II\nu}, P_{II\nu}^{P_{II\nu}}$ given by Bukowsky.
For the relevant \( \sigma \), \( \mathcal{P}^\sigma \subseteq M_k^{1}(k_0^\sigma) + \mathcal{P}_k \), and \( \mathcal{P}^\sigma \subseteq \mathcal{P}^{\sigma'} \) for \( \nu \leq \nu' \) (so that the \( \mathcal{P}^\sigma \) stabilize).

We have \( L_0 = P \). We leave the details to the reader.

The proof of Claim 21, p.64, shows:

**Claim 24.** In \( V \), \( L_0 = P \) is iterable w.r.t. trees which are extendable from the \( P \)-sequence.

There is a more general notion of iterability of \( L_0 \), though.

Choose \((\omega_0, \nu_0, \omega, (\rho P^\nu), \delta_0 \mu_0)\), the map \( (\rho P^\nu) \delta_0 \mu_0 \) is intertranslatable with

\[
\{ (\bar{T}, \Sigma_{\omega_0}^{\nu_0} \mu_0 (\bar{T})) : \bar{T} \in \omega_0 \delta_0 \mu_0 \text{ is a tree of limit length on } \omega_0 \delta_0 \mu_0 \}
\]

so that we reorganized \( L_0 \) as a "premouse" of the form \( L \models \forall E, \exists S \), where
\[ \mathbf{\check{E}} \text{ is a sequence of extenders,} \]

\[ \mathbf{\check{E}} \upharpoonright \kappa_0^+ \sigma^0 = \mathbf{E} \sigma^0 \upharpoonright \kappa_0^+ \sigma^0, \]

\[ \mathbf{\check{E}} \upharpoonright \kappa_0^+ \sigma^0 \alpha = \{ \mathbf{E} \sigma^0 \cap \mathbf{P} \mid \nu : \nu > \kappa_0^+ \sigma^0, \]
\[ \text{crit}(\mathbf{E} \sigma^0) > \kappa_0 \}, \text{ and} \]

\[ \mathbf{\check{S}} = \{ S_\nu : \nu \in \{ \kappa_0^+ \sigma^0 \} \cup \{ \nu : \text{crit}(\mathbf{E} \sigma^0) = \kappa_0 \} \}, \]

\[ S_\nu = \{ (\mathbf{\check{E}}, \check{S}^\nu) : \mathbf{\check{E}} \in \mathbf{P} \mid \mathbf{\check{E}} \nu \text{ is a} \]
\[ \text{tree of limit length on } \sigma^0 | \kappa_0^+ \sigma^0 \} \).

Here, \( \mathbf{\check{P}} \mid \nu = \mathbf{\check{E}} \upharpoonright \kappa_0^+ \sigma^0 \), \( \mathbf{\check{P}} \check{\nu} = \)
\[ (\mathbf{\check{P}} \check{\nu}; (\mathbf{E} \check{S})_\nu) \).

Claim 24 says that \( \kappa_0 \) is iterable w.r.t. \( \mathbf{\check{E}} \)
and its images.

However, each \( S_\nu \) as above is intertranslatable over \( \mathbf{\check{P}} \check{\nu} \) with a (long) extender, namely the canonical map

\[ \check{\pi}_\nu : \sigma^0 | \kappa_0^+ \sigma^0 \rightarrow (\sigma^0)^+, \]
where \((\mathcal{U}_0)\) is the direct limit of all non-dropping (in the main branch) \(\Sigma_{\omega_1}\delta^{\omega_0}\) club sets of \(\mathcal{U}_0\) which \(P|\lambda^P\) can see. By normalization (Schlichtening-Steel), \(\pi_2\) as above is given by a normal iteration of \(\mathcal{U}_0\) according to \(\Sigma_{\omega_1}^{\omega_0}\).

We may then "take the ultrapower" of \(\mathcal{U}_0\) by \(S\) as \(\text{ult}(\mathcal{U}_0; \pi_2)\). As \(\pi_2\) is given by a normal tree on \(\mathcal{U}_0\), \(\text{ult}(\mathcal{U}_0; \pi_2)\) is given by a normal iteration of \(\mathcal{U}_0\) via a tree living on \(\mathcal{U}_0\).

The more general notion of a iteration of \(\mathcal{U}_0\) is then given by allowing \(\text{ult}(\mathcal{U}_0; \pi_2)\) in course not only for \(\mathcal{U}_0\) itself but also for iterates of \(\mathcal{U}_0\).
The key fact is the following. Let $I$ be a interval of $\omega$ (in the generalised sense) and $\mathcal{M}_{I}$ be its model $\mathcal{M}_{I}$. Say $[0, \omega]_{I}$ doesn't drop.

We claim that if $S_{I}^{\mathcal{M}_{I}} \neq \emptyset$, then

$$S_{I}^{\mathcal{M}_{I}} = \{ (T, \Sigma) \in \prod_{\mathcal{M}_{I}}(\omega_{\omega}) | \Theta_{T}^{\mathcal{M}_{I}} \} \quad : \quad T \in \mathcal{M}_{I} \times \omega_{\omega} $$

is a tree of limit types on $\prod_{\mathcal{M}_{I}}(\omega_{\omega} \mid \omega_{\omega})$.

By interval coherence, which is $1^{\text{st}}$-order, it suffices to prove this for $z \in \text{ran}(\pi_{0x}^{I})$, say $z = \pi_{0x}^{I}(\omega)$. Let $U^{\omega}$ be the tree on $\omega_{\omega} \mid \omega_{\omega}$ which gives rise to $\pi_{z}^{I}$, $\text{eh}(U^{\omega}) = z$, $U^{\omega}$ being definable over $\mathcal{M}_{I}$; $S_{I}^{\mathcal{M}_{I}}$ can be read off from $\sum_{\omega_{\omega} \mid \omega_{\omega}}(U^{\omega})$.

Let $U^{\omega}$ be the tree on $\prod_{\mathcal{M}_{I}}(\omega_{\omega} \mid \omega_{\omega})$ which gives rise to $\pi_{z}^{I}$ ($\pi_{z}^{I}$ the map induced by $S_{I}^{\mathcal{M}_{I}}$), $U^{\omega}$ being definable on $\prod_{\mathcal{M}_{I}}(\omega_{\omega})$ in the same manner as $U^{\omega}$ was definable on $\mathcal{M}_{I}$, $\text{eh}(U^{\omega}) = z$; $S_{I}^{\mathcal{M}_{I}}$ can be read off from $\sum_{\pi_{0x}^{I}(\omega_{\omega} \mid \omega_{\omega})}$. Inductively, $\pi_{0x}^{I}(\omega_{\omega} \mid \omega_{\omega})$ is a $\omega_{\omega}$-iterate of $\omega_{\omega} \mid \omega_{\omega}$. 
Notice that $\delta_0^{\mu_0}$ is a continuity point of all the relevant framed maps, so that $\frac{\pi_I}{\omega_0}$ is continuous at $\vec{u}$.

\[ \Sigma_{\mu_0}^{\mu_0} \text{ is the framed map at } \mu_0 \]

\[ \Sigma_0^{\mu_0} (U) \]

\[ \mathbb{M} \rightarrow \mu_0 \rightarrow \mathbb{U} (\mu_0 ; \frac{\pi_I}{\omega_0} (\mu_0) \rightarrow \mathbb{R}_0 \]

\[ \Sigma_0^{\mu_0} (U) \]

\[ \mathbb{R}_0 \]

We may pull back $\Sigma_0^{\mu_0} (U)$ to produce a cofinal branch $\sigma$ through $U'$ s.t. the limit model $\mu_0$ can be embedded into $\mathbb{R}_0$ ($\mathbb{R}_0$ as in the diagram). By branch condensation for $\Sigma$,

\[ c = \Sigma_0^{\mu_0} (U) \]

\[ \text{But then } \Sigma_0^{\mu_0} (U) \text{ is the cofinal} \]
branch then \( \Sigma \) given by lifting up
\[ \sum_{m \in \mathbb{N}} (u^m) \], which is also the branch
given by \( \sum_{m \in \mathbb{N}} u^m \). Therefore, \( \pi \) moves to

correctly.

We have shown that \( \Sigma \) is iterate in the
generalized sense.

Another correction. Possibly, Claim 13 on p. 50
is not quite right; at least its proof seems
suspicous. *) Let us replace Claim 13 by the
following.

Claim 13'. \( \Sigma = L[\mu_0, \rho \uparrow \rho^+] = HOD_{\Sigma} \mathcal{E} \)

where \( \Sigma \) is defined as follows.

Let \( L[\varepsilon] \) be an extensible model, and let \( \kappa \) be
a regular cardinal of \( L[\varepsilon] \). Let \( g \) be
Co\( _1 (\varepsilon, \kappa) \)-generic on \( L[\varepsilon] \).

Let us assume that \( \kappa \) is a "weak cutpoint.”

*) We thank F. Schlutzenberg for pointing this out to us.
of LEE) in the sense that there is no $E_\nu$ of LEE's extendible sequence with $\nu > \kappa$ and $\text{crit}(E_\nu) < \kappa$ (where $\text{crit}(E_\nu) = \kappa$ is being allowed).

Let us define $\tilde{E}_\zeta = (\tilde{E}_\zeta, \zeta > \kappa, E_\zeta$ ache), where $\tilde{E}_\zeta$ is the extendible or ideal extendible (cf. Claverie's Ph.D. thesis) defined by

$$\forall \zeta \in \tilde{E}_\zeta, a \iff \exists \gamma \in \text{L[E]}[\gamma] \land \gamma \in \text{P}(\text{crit}(E_\zeta))^\kappa \land \exists X < Y \in E_\gamma \land Y \in E_\zeta$$

(Then $\tilde{E}_\zeta$ is an extendible iff $\text{crit}(E_\zeta) > \kappa$ and it is an ideal extendible iff $\text{crit}(E_\zeta) = \kappa$.)

Let LEE, LEE' be two extendible models. We say that LEE, LEE' are intertranslatable above $\kappa$ iff $\kappa$ is a regular cardinal and a weak cutpoint in both LEE and LEE' and for some (all) $g \in \text{Con}(\kappa, \omega)$-jump / LEE $\lor$ LEE'.

$$\tilde{E}_\zeta = \tilde{E}_\zeta'$$
(It then follows that if $L[E][g] = L[E'][g]$, $E$ is definable from $E|\kappa$ and $E_g$ inside $L[E][g] = L[E'][g]$, and conversely $E'$ is definable from $E'|\kappa$ and $\hat{E}_g$ inside the same model.)

We write $\mathcal{E}$ for the "set" of all $E$ s.t. $L[E]$ "$\kappa$ is the least inner model with $\delta_0 < \kappa$, $\delta_i$ wooden, and $\kappa_i$ strong, i.e. $\{\gamma_0\}$, and $\kappa$ sufficiently iterable," and there is some $\gamma < \kappa_0$ s.t. $L[E]$, $M = M_{\gamma} \cap \mathcal{E}$ are intranslatable above $\kappa$ (in particular, $\kappa_0 = \kappa_0$, $\delta_0 = \delta_0$, $\kappa_0 = \kappa_0$). (Notice that if $E \in \mathcal{E}$, then $E \in L[E]$ (i.e., $E$ as being definable inside $L[E]$) is equal to $E = E^M$.)

Proof of Claim 13': The proof of [VMM, Claim 2.10] shows that if $E \in \mathcal{E}$, then the $\omega_1$-system of $L[E]$ has equally many points.
in common with the \( w_0 \) of \( M \).

This shows that \( \langle w_0, P^+, R^+ \rangle \subseteq HOD \).

Now let \( X \) be a set of ordinals in \( HOD \). Say \( \xi \in X \) iff
\[
\forall \alpha < \Omega \left( \forall \beta < \omega \left( P^{\xi}(\beta, \alpha) = \xi_\beta^{\xi} \right) \right).
\]

Then \( \overline{P} \) (or rather its extend sequence) is in \( \mathcal{E} \), so that \( \mathcal{E}^{\xi} = \mathcal{E} \). Hence \( \xi \in X \) iff
\[
\forall \alpha < \Omega \left( \forall \beta < \omega \left( P^{\xi}(\beta, \alpha) = \xi_\beta^{\xi} \right) \right) \iff \xi \in \mathcal{E}.
\]

We have shown that \( X \in \mathcal{L} \).

\( \vdash \) (Claim 13')
Claim 13' may be used in a nice way to show that the reorganization of $\nu_0$ along the lines of pp. 83-85 works.

The point is just that now, writing $M = M_{\text{susw}} \subseteq L[E]$, if $g$ is any $\text{Col}(w, \geq i_0)$-generic filter of $M$, then $\tilde{E}_g$ (def. from $E$ as on p. 91) is OD in $M[g]$, as $\tilde{E}$ is the common value of all $\tilde{E}_g$ for $E' \subseteq E$. But then a straightforward induction gives that all the models of the construction from pp. 83-85 are in $\text{HOD}^M_{\text{Col}(w, \geq i_0)}$, hence by Claim 13' they are all in $\nu_0$. This construction thus gives a reorganization of $\nu_0$.

To be cont'd.