today: pg. 57: double sci + firm analyses.

Let: \( P \) is not proper.

\[ \text{pg}: \text{ suppose that } x \in X \subseteq \mathbb{R}, \text{ etc.} \]

may find \( G \) so \( P \)-f.p. in \( V \) s.t.

\[ X \cap G \cap H = X \cap H \]

where \( x = X \cap \eta \).

So if \[ \eta \in \eta \]

\[ N \cap G = X \cap G \subseteq H \cap \eta \cap \)

\[ \eta \cap (H \cap \eta \cap \eta \cap \eta) \cap \eta \cap \eta \cap \eta \cap \eta \]

get by \( G \), then

\[ M = \eta \cdot (H \cdot \eta \cdot \eta \cdot \eta \cdot \eta) \cap \eta \cap \eta \cap \eta \cap \eta \cap \eta \]

\[ = (H \cdot \eta \cdot \eta \cdot \eta \cdot \eta) \cap \eta \cap \eta \cap \eta \cap \eta \cap \eta \]

so if \( H \) is ex., then \( H \) is in \( V \) and is unique to \( H \)

in \( V \) which is not in a boundary.
recall:

\[ \text{THM (double-shRN)} \quad TFAE \]

1. \( N_{\mathbb{R}_1} \) is precipitous + \( \mathfrak{p} \) for \( \mathfrak{b} \), \( \mathfrak{p} \in \text{des-pmax} \)

2. \( \text{(*)} \)

proof: \( (2) \Rightarrow (1) \) (magidor–foreman–melah)

let \( \mathcal{M} \) be an antichain in \( (N_{\mathbb{R}_1})^+ / N_{\mathbb{R}_1} \), and let \( S = S(\mathcal{M}) \) be the corresponding scaling forcing.

in \( X \subseteq H_\alpha \)

in \( \mathcal{G} \subseteq X \), \( \mathcal{G} \) is \( \mathfrak{p} \)-preserving, and \( \mathcal{G} \) is \( S \)-preserving.

\[ \chi_{\mathcal{G}} \prec H_\alpha \setminus \mathcal{G} \]

(also \( \chi_{\mathcal{G} \cap H_\alpha} \prec H_\alpha \)).

thus \( \alpha = X \cap \mathbb{R}_1 = \chi_{\mathcal{G} \cap \mathbb{R}_1} \).
let \((C, e: s_1 \rightarrow \alpha_1)\) be \(\pi^n\) be \(G\)

in \(CC_u\), is \(\alpha_1\), ad for all \(\beta \in C\),

there is some \(p < \beta\) s.t. \(\beta \in e(p)\).

hence \(\alpha \in C\), so \(\alpha \in e(p)\), for \(p < \alpha\).

so \(e(p) \subseteq X \subseteq e(p)\).

by absoluteness, in \(V\), there is some

\(Y \supset X\), \(X \times Y \not\subseteq H\), \(\forall\alpha \omega, x \in X\), \(\tau \alpha\),

\(e(p) \subseteq Y\), i.e.,

\(\exists s \in \alpha \forall y \in S\).

using this, we may build a tower of structures

whose union is $Z$

\[ Z \subseteq H \omega, \quad \alpha = Z \omega_1 \]

so if \(\beta \in Z\) is an ordinal, then

\(\exists s \in \alpha \forall y \in S\), hence $Z$ is "self generous," i.e., if
\[ N \equiv 2 \times H_X \]

and \( u \) is the \( N \)-neighbor \( n \) such that \( u \in \cup_{v \in N}

\( \left( NS_m \right) \)

may be fixed \( S \in (NS_m)^+ \). As the outcome,

picked \( x \) s.t. \( x = x_n \in S \).

then can be had \( S \vdash \text{Un}(v, \xi) \) is ill-

\[ v \]

\[ S \vdash \text{Un}(N, \xi) \text{ ill-ref.} \]

\[ \text{(2) } \Rightarrow (1) \]

\textbf{Remark:}

can step up the

the larger pie also shows that

\( (+) \Rightarrow \{ x \times H_x : x \text{ is self-m.} \} \)

is stach.

it is easy to verify that this conclusion is

equivalent to the precipitation of \( N S_m \).

can be: \( N S_m \) or \( (=) \{ \ldots, 3 \text{ is club} \). \)
\[ y' \text{'s ends} \]

\textbf{General: TFAE}

1. \( \mathcal{N}_\omega \) is \{ \text{precession} \}

2. \( \{ x < \mathcal{H}_\omega : x \not\in y_{-\omega} \} \) is \{ \text{stationary} \}

\( \text{If } (1) \Rightarrow (2), \text{by double-nach : we need a stationary of chang's conjecture :} \)

\text{def. } \mathbf{CC}^{**}:

\( \forall \omega : \omega > \omega \geq \omega_2 , \text{ if} \)

\[ x < \mathcal{H}_\omega + \alpha \in [\omega]^{\omega} \]

then there is some \( y, \)

\[ x < y < \mathcal{H}_\omega \]

\( y \cap \omega = x \cap \omega \quad \text{and} \quad \exists \beta \in y \cap [\omega]^{\omega} \ni \beta > \alpha, \)
\[ \text{Proof: } \forall \gamma = \omega_2, \forall \omega_1 \text{ is ordinal of } \mathbb{C}^* . \]

\[ (1) \Rightarrow \mathbb{C}^{**} : \]

\[ \text{fix } \Theta = \Theta \geq \omega_2, \text{ fix } X < H_\Theta, A \in \lambda \omega, \]

we have a supra for \( \gamma \) for adding \( (M_i, \pi_i: i \leq \omega_1) \), given by \( G \).

So all \( h_i, i < \omega_1 \) are \( \text{clo} \), \( M_{\omega_1} = H_\Theta \).

In part, \( a \in M_{\omega_1} = \text{Hull}_{\mathcal{H}_\Theta}(\omega_1 \cup \text{ran}(\pi_{\omega_1})) \),

so \( a = t^{\mathcal{H}_\Theta}(\gamma_0, y), \quad \gamma_0 < \omega_1, \quad y \in \text{ran}(\pi_{\omega_1}) \).

Set \( b = \left( \bigcup \{ t^{H_\Theta}(\gamma, y) : \gamma < \omega_1, \quad t^{H_\Theta}(\gamma, y) \leq \gamma_1 \} \right) \)

\[ \bar{b} \leq \gamma_1, \quad b \in \mathcal{V}_0 \text{ is a } \mathcal{V}_0 \text{-set. In fact,} \]

\[ b \in \text{ran}(\pi_{\omega_1}), \quad a \text{ is defined for } \]

\[ \text{far beyond } \text{ran}(\pi_{\omega_1}). \]
\[ M = \{ x \in \mathbb{C}^n : x \neq 0 \} \subset \mathbb{C}^n. \]

If \( x \in X \), then by construction

\[ b \in \mathbb{C}^n \subset \text{ran} (\pi_{\omega_1}). \]

Now by absolute, \( x \perp y \) there is \( \gamma \)

\[ X \perp y \subset \mathbb{C}^n \quad \gamma \omega_1 = x \omega_1. \]

\[ \exists \beta \in \mathbb{R}^\omega \setminus \{ 0 \}, \beta > \alpha. \]

Thus, when (1) \( \leftrightarrow \mathbb{C}^{\star \star} \).

def. \( S \subset \mathbb{C}^\omega \) is semi-infl. if

\[ \left( \bigcup_{x \in S} \{ y \in \mathbb{C}^\omega : y \perp x \} \right) \text{ is stat. } \quad x = y. \]

\[ SSR(\mathbb{C}[\mathbb{C}^\omega]) \equiv \text{semi-infl. rep}. \equiv \]

\[ S \subset \mathbb{C}[\mathbb{C}^\omega] \text{ semi-infl.} \Rightarrow \exists w \in A (\mathbb{C} = N \land \]

\[ S \cap \mathbb{C}^{\star \star} \text{ semi-infl. in } \mathbb{C}[\mathbb{C}^{\star \star}]. \]
\[
cc^* \rightarrow s_{\not\exists}(\exists^x) \forall \lambda \equiv \omega_2.
\]

**Proof:** Let \( S \subseteq \exists^\omega \) be \( \Delta^0_2 \).

\[ T = \{ y \exists^x : y \in \exists^\omega, x \in S \} \text{ s.t.} \]

Suppose, f.a. \( w \in T \), \( \bar{w} = \eta \).

\[ S_w = \{ y \exists^x : y \in \exists^\omega, x \in S \cap \exists^w \} \]

is not s.t. when \( y \exists^w \). \( f_w : \exists^w \rightarrow w \).

Let \( a \in \eta \). \( \exists \eta \) \( X < H_\eta \), \( (f_w : w \in \exists^w, \bar{w} = \eta) \in X \)

s.t. \( X \cap \eta \in T \).

\( a = \omega \cup (X \cap \eta) \).

Then

\[ X \lessdot \gamma < H_\eta, \quad \gamma \in X \]

s.t. \( \eta \in \gamma \) for some \( \gamma \in \exists^\omega \).

If the def. of \( T \), then is some \( x \in S \),

\[ x \in X \cap \eta = \gamma \cup \eta \]
by the choice of $f_b : [b]^\omega \to b$,
\[ \{ z \in [b]^\omega : z \text{ closed } \downarrow f_b \} \cap S_b = \emptyset. \]

Thus, $y_{nb} \in [b]^\omega$, $y_{nb} \supseteq x \in S$, $x \in [b]^\omega$

\[ (y_{nb})_n \omega_1 = y_{n \omega_1} = x_\omega \omega_1 = 2^n \omega_1, \]
if $\exists x$, $\exists Xn\lambda$, so $\exists b > Xn\lambda$ and $\exists \epsilon \forall Y$

So $y_{nb} \in S_b$.

but also $f_b \in Y \ (a, b \in Y)$, so

$y_{nb}$ is closed $\downarrow f_b$.

whence!

\[ \text{cc}^{**} \Rightarrow \text{ssr} ([a]^\omega) \]

$h_e (\text{numb})$; $\forall \text{ ssr} ([a]^\omega) \Rightarrow (\dagger)$
\( \forall p \in P \) \exists \text{ winning strategy } s_t \ni

Fix \( s \). Assume the win \( X \) s.t.

\[ X < V_n \]

and this is no \( q \leq p \) and is \( s_t \)-win. \( X \)

is stationary.

So \( \mu \) is a \( \mathcal{W} \in V_0 \) of the \( \lambda \) s.t. \( \mathcal{W} < V_n \)

the \( \sigma \) of \( X < V_n \), \( X = W \) s.t. \( \mu \) is

no \( q \leq p \) s.t. \( X \) is separated. in \( [\mathcal{W}] \).

Let \( f : \omega \rightarrow W \) \mu, \( \text{ in } G \) be \( P \rightarrow P \) \mu V.

Pick \( \alpha \) s.t. \( f''_{\alpha} \mu = \alpha \),

\[ f''_{\alpha} \subseteq X \], \( \text{ s.t. } X < V_n \), \( X \subseteq \mathcal{W} \),

\[ \text{ no } q \leq p \text{ s.t. } X < \mathcal{W} \].

\[ f''_{\alpha} \rightarrow f''_{\alpha} < \mathcal{W} \].

\[ f''_{\alpha} [G] \mu = \alpha \]

but then for \( q \in G \) is \( X \) s.t. \( X < \mathcal{W} \).