nam, cont'd

$$\Omega = \{ A : A \leq_w F \upharpoonright \mathbb{R} + F \text{ is a mouse operator} \}$$

Succ. step : $F \longmapsto M_1^{F,\#}$

lim step :

Ω

$F \to \vdash \quad \underset{\sim}{\Delta}$

<u>case:</u> $\mathrm{cf}(\overbrace{w(\underset{\sim}{\Delta})}^{\text{wadge rank}}) = \omega$

easy ; take join of $(F_i : i < \omega)$, where $\sup w(F_i) = w(\underset{\sim}{\Delta})$.

<u>case</u> : $\mathrm{cf}(w(\underset{\sim}{\Delta})) > \omega$.

<u>Subcase 1.</u> inadmissible case + solovay seq. succ. lgth

there is a "companion model" M s.t. $\underset{\sim}{\Delta} = \mathcal{P}(\mathbb{R}) \cap M$. (e.g., $M = J_\kappa(\mathbb{R})$, where κ is inadmissible.)

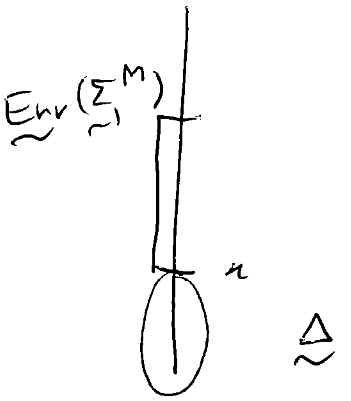
look at $\underset{\sim}{\Sigma}{}^M_1$.

Φ "diagonal operator" that captures $\underset{\sim}{\Sigma}{}^M_1$.

now use hypo to get $M_1^{\Phi,\#}$

<u>subcase 2.</u> induction-like case + Solovay seq. succ. length

(gap case)



$\underset{\sim}{\Sigma}{}^M_1$: induction-like :

i.e., closed under

$\exists^{\mathbb{R}}$, $\forall^{\mathbb{R}}$.

look at $\mathrm{Env}(\underset{\sim}{\Sigma}{}^M_1)$.

need a scale on $\underset{\sim}{\Pi}{}^M_1$ whose individual norms are in $\underset{\sim}{\mathrm{Env}}(\underset{\sim}{\Sigma}{}^M_1)$;

transfer th. gives you det$(\underset{\sim}{\mathrm{Env}}(\underset{\sim}{\Sigma}{}^M_1))$ here.

[rmk.: e.g. going from $\theta_0 = \theta$ to $\theta_0 < \theta$ is an instance of this subcase.]

then can build $\mathcal{I}$ that "captures" this ~~??~~ scale (this uses background hypo).

<u>Subcase 3.</u> Solovay sequence of $\underset{\sim}{\Delta}$ is of lim length.

$$\theta_0^{\underset{\sim}{\Delta}} = \sup \{ \theta_0^{L(A,\mathbb{R})} : A \in \underset{\sim}{\Delta} \}$$

$$\theta_{\alpha+1}^{\underset{\sim}{\Delta}} = \sup \{ \theta_{\alpha+1}^{L(A,\mathbb{R})} : A \in \underset{\sim}{\Delta} \}$$

λ lim, $\theta_\lambda^{\underset{\sim}{\Delta}} = \sup_{\alpha<\lambda} \theta_\alpha^{\underset{\sim}{\Delta}}$.

analyze hod limits of $\underset{\sim}{\Delta}$.

<u>th.</u> ~~PFA~~.

1) Con (PFA) $\longrightarrow$
Con ($\exists$ non-domestic mouse)

2) Con (~~ZF~~ PFA + $\exists$ Woodin cardinal)

$\longrightarrow$ Con (there is a Woodin limit of Woodins)

pf.: 2) deny. ~~blah~~ let δ be ~~the~~ the least Woodin of V.

let (M_ξ, N_ξ : $\xi \leq \delta$) be the L[E] constr. this works by Neeman.

let $N = N_\delta$. let $S = S(N)$,

where $S(N) = \bigcup \{ M : M$ sound, $\rho_\omega(M) = \delta$, M is ctbly. iterable $\}$.

PFA $\Rightarrow$ cf(o(S)) $< \delta$.

pf.: PFA $\rightarrow \neg\square(\delta)$.

so if cf(o(S)) $= \delta$,

have an unthreadable coh. sq. of lngth δ. $\not\!\!\xi$.

by cong argmt, let $\theta >> \delta$.

take

$$X \prec H_\theta,$$

${}^\omega X \subset X$, $X \cap \delta = \kappa < \delta$, and letting

$$\pi : M_X \longrightarrow H_\theta,$$

$$\pi^{-1}(S) = \mathcal{N} \mid \kappa^{+\mathcal{N}}.$$

so $\mathcal{N} \models$ "κ is woodin"

by elementarity.

hence get that δ is a lim of woodin

cardinals. $\dashv$