farmer 2.

let α_x = least α s.t.

R, S are not (α, x)-compatible

$\alpha_x = \infty$ if no such α

α_y likewise.

case 1. $\alpha_x = \alpha_y = \infty \rightsquigarrow$ stop.

note. case 1: we're done.

(easy induction on $\alpha < lh(\mathbb{C}_x^R), lh(\mathbb{C}_x^S)$,

shows $N_\alpha^{\mathbb{C}_x^R} = N_\alpha^{\mathbb{C}_x^S} \subset R \cap S$:

if $N_{\alpha+1}^{\mathbb{C}_x^R} = (N_\alpha; E)$, $N_{\alpha+1}^{\mathbb{C}_x^S} = (N_\alpha; E')$

$F = F_\alpha^{\mathbb{C}_x^R}$, $G = F_\alpha^{\mathbb{C}_x^S}$,

then $\nu_E < OR^{N_\alpha} \leq \alpha \leq \gamma_\alpha' < \theta$

$\Rightarrow \quad F \restriction N_\alpha \times \nu_E^{<\omega} = G \restriction N_\alpha \times \nu_E^{<\omega}$

$\Rightarrow \quad E = E'$.

case 2a. $\alpha_x < \alpha_y$.

let
$$E^{\nu}_{\beta} = F^{\mathbb{C}^R_x}_{\alpha_x}$$
$$E^{w}_{\beta} = F^{\mathbb{C}^S_x}_{\alpha_x}.$$

case 2b. $\alpha_y < \alpha_x$.

similar.

case 3. $\alpha_x = \alpha_y < \infty$.

then let $E^{\nu}_{\beta} \in \{ F^{\mathbb{C}^R_x}_{\alpha}, F^{\mathbb{C}^R_y}_{\alpha} \}$

of minimal index,

$E^{w}_{\beta} \in \{ F^{\mathbb{C}^S_x}_{\alpha}, F^{\mathbb{C}^S_y}_{\alpha} \}$ of min. index.

let $\alpha_\beta = \alpha = \min(\alpha_x, \alpha_y)$.

claim 1. ~~the~~ ν, w are normal.

in fact, $\beta < \gamma \Rightarrow \alpha_\beta < \alpha_\gamma$.

$$(\, str(E^{\nu}_{\beta}) = \aleph'^{M^{\nu}_{\beta}}_{\alpha_\beta} \,).$$

claim 2. comparison stops.

prf. of claim 1:

if not, let (β, γ) be the min. counterexample.

$\beta < \gamma$, $\alpha_\gamma \leq \alpha_\beta$.

facts. note if $F = F_\alpha^{\mathbb{C}_x^P} \neq \emptyset$, then

(1) $P \mid \aleph_\alpha^P \not\models ZFC$.

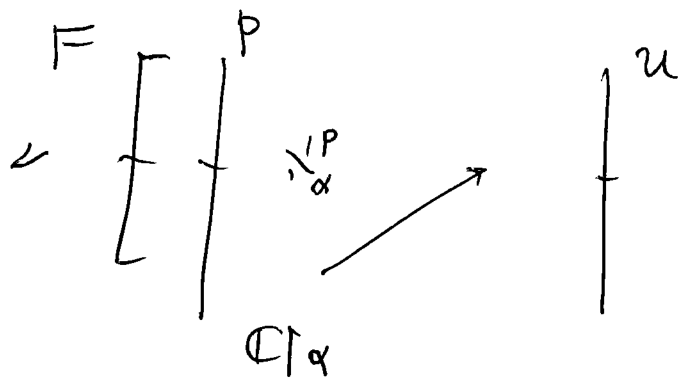
(2) let $U = \mathrm{Ult}(P; F)$. then

$\mathbb{C}_x^U \restriction \alpha = \mathbb{C}_x^P \restriction \alpha$,

$F_\alpha^{\mathbb{C}_x^U} = \emptyset$

$F_\alpha^{\mathbb{C}_y^U} \neq \emptyset$ iff $F_\alpha^{\mathbb{C}_y^P} \neq \emptyset$ +

$lh(F_\alpha^{\mathbb{C}_y^P}) < lh(F)$.



$\aleph_\alpha^U = \aleph_\alpha^P$.

$\aleph_{\alpha+1}^U = lh(F)$.

$$N_\alpha^{\mathbb{C}_x^P} = N_\alpha^{\mathbb{C}_x^U}.$$

(3) get similar facts of agreements between $\mathbb{C}_x^{M_\beta^\nu}$, $\mathbb{C}_x^{M_\beta^w}$

$$\mathbb{C}_x^{M_\beta^\nu} \restriction \alpha_\beta = \mathbb{C}_x^{M_\beta^w} \restriction \alpha_\beta.$$

go back to $\alpha < \gamma$, $\alpha_\gamma \leq \alpha_\beta$.

harder case: $\alpha_\gamma = \alpha_\beta = \alpha$.

$E_\beta^\nu = F$ $\qquad F' = E_\beta^w$

$$\left[\; \lambda_\alpha^{M_\beta^\nu} = \lambda_\alpha^{M_\gamma^\nu} \; \right]$$

let $F = E_\beta^\nu$

supp. $x = x_\beta^\nu$,

where:

$$x_\beta^\nu = \begin{cases} x & \text{if } E_\beta^\nu \text{ chosen } \not\ni \text{ } (\alpha, x)\text{-incompatible} \\ y & \text{o/w} \end{cases}.$$

x_β^w = likewise.

supp. $E_\beta^\nu \neq \emptyset \neq E_\beta^w$.

Then $\alpha_\gamma = \alpha_\beta$.

$\Rightarrow \nu(E^\nu_\gamma) = \lambda'^{M^\nu_\gamma}_\alpha = \lambda'^{M^\nu_\beta}_\alpha$.

let $G = E^\nu_\gamma$.

note $G = \cancel{E}\ F_\alpha^{\mathbb{C}_y^{M^\nu_\gamma}}$, as $F_\alpha^{\mathbb{C}_x^{M^\nu_\gamma}} = \emptyset$.

$\Rightarrow G = F_\alpha^{\mathbb{C}_y^{M^\nu_\beta}}$.

but $\mathrm{lh}(G) < \mathrm{lh}(E^\nu_\beta)$.

$\Rightarrow M^\nu_\beta, M^\mu_\beta$ were (α, y)-compatible

$\downarrow$

$\Rightarrow G' = F_\alpha^{\mathbb{C}_y^{M^\mu_\beta}} \neq \emptyset$

+ G, G' are compatible over

$(M^\nu_\beta \cap M^\mu_\beta) \times \theta^{<\omega}$

$\theta = \min(\mathrm{lh}(G), \mathrm{lh}(G'))$.

$\Rightarrow$ if $G' \in \mathbb{E}^{M^\mu_\gamma}$, then

G, G' are min compatible

w.r.t. M_γ^{ν}, M_γ^{w} $\lightning$

$\Rightarrow \; lh(E_\beta^{w}) \le lh(G')$.

<u>Subcase 1.</u> $G' = F'$.

G compatible with $G' = F'$
$\rightsquigarrow$ adding an extender, i.e.
$F' = F_\alpha^{\mathbb{C}_x^{M_\beta^{w}}}$

~~[illegible]~~

So F' adds E to $N_\alpha^{\mathbb{C}_x^{M_\beta^{w}}} = N_\alpha^{\mathbb{C}_x^{M_\beta^{\nu}}}$

$\Rightarrow$ G adds E also to

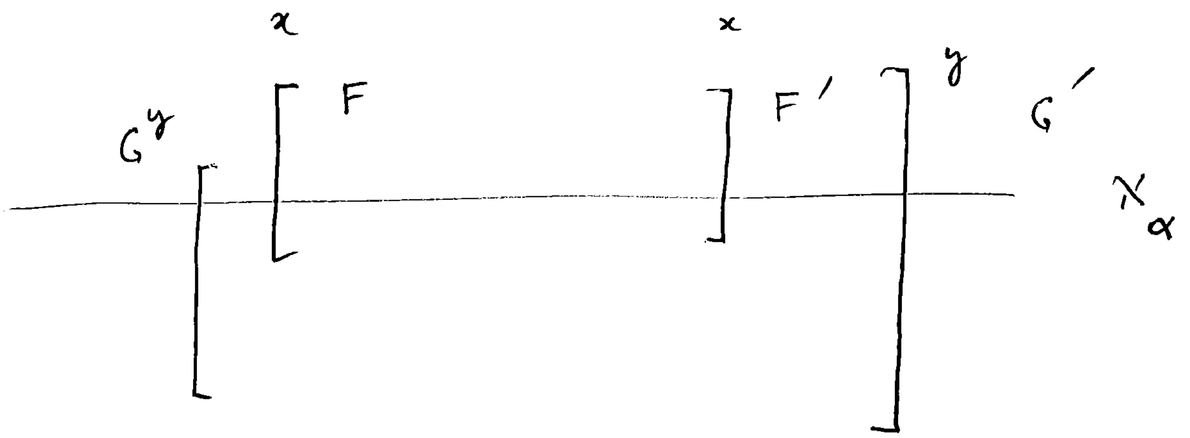
F' induces E

$\Rightarrow$ G' induces E on $N_\alpha^{\mathbb{C}_x^{M_\beta^{\nu}}}$

$\Rightarrow$ G works as background

$\Rightarrow \; lh(F_\alpha^{\mathbb{C}_x^{M_\beta^{\nu}}}) \le lh(G)$ $\lightning$

<u>Subcase 2.</u> $lh(F') < lh(G')$.



stage β

ν ω

let $\kappa = \operatorname{crit}(G) = \operatorname{crit}(G')$.

let $A = \{\mu < \kappa : F_\mu^{\mathbb{C}_x^{M_\beta^\omega}} \neq \emptyset\}$

$= \{\mu < \kappa : F_\mu^{\mathbb{C}_x^{M_\beta^\nu}} \neq \emptyset\}$.

$\Rightarrow A \in M_\beta^\nu \cap M_\beta^\omega$.

$F' \in \operatorname{ult}(M_\beta^\omega, G')$

$\Rightarrow \alpha \in i_{G'}^{M_\beta^\omega}(A)$.

$\alpha < \theta = \min(\operatorname{lh}(G), \operatorname{lh}(G'))$.

by compatibility,

$$\alpha \in i_G^{M_\beta^\nu}(A)$$

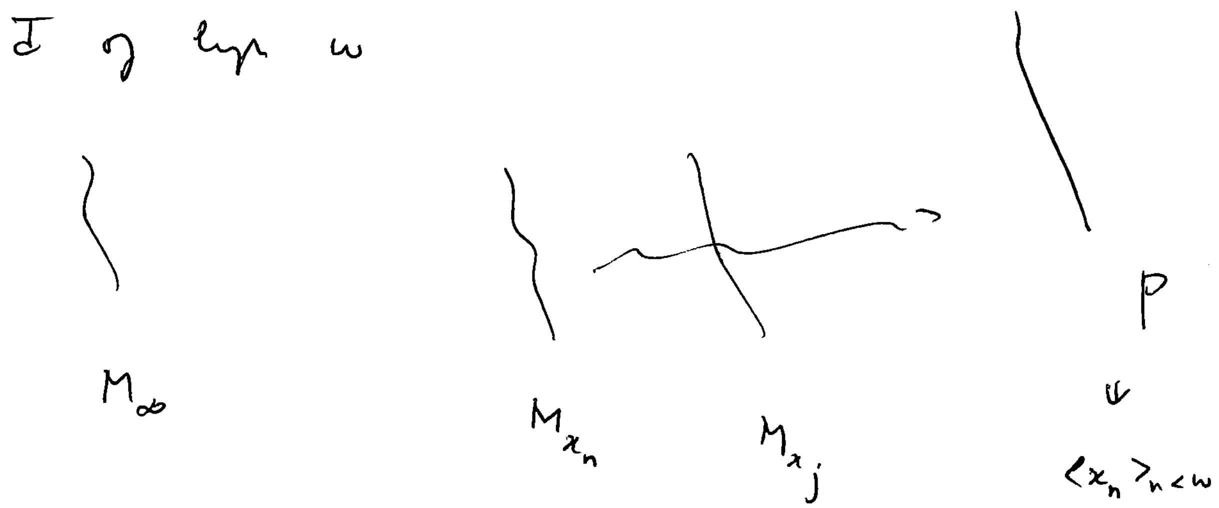
$$\Rightarrow F'' = F_\alpha^{\mathbb{C}_x^{un(M_\beta^\nu;\, G)}} \neq \emptyset .$$

$\Rightarrow lh(F'') < lh(G) < lh(F)$, contradicts the minimality of F.

Semiscale (Π^1_3).

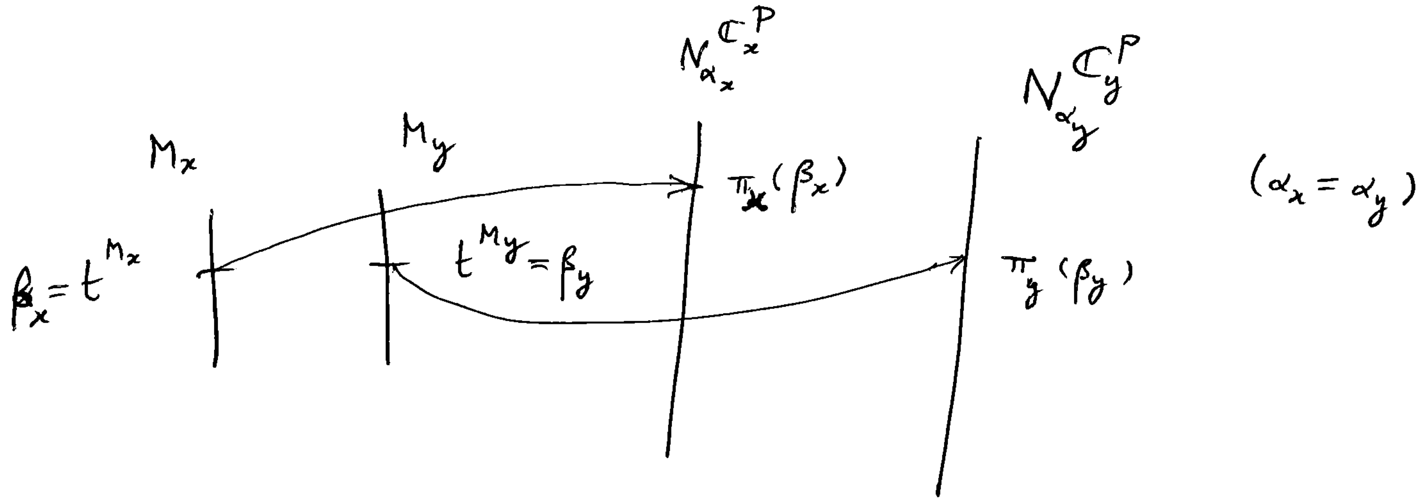
was M_∞ iterate.

$\mathcal{I}$ of type ω



<u>plan:</u> define norms analyzing how finite iterates of M_x, M_y lift to trees on background universes $\mathbb{P}$.

<u>depth 1 norms.</u> let P be x, y-good.



for a term t,

define norm $\varphi_t(x) = \pi_x(t^{M_x})$.

here,

$$\pi_x : M_x \longrightarrow N_{\alpha_x}^{\mathbb{C}_x^P}.$$

---

for terms t s.t. t^{M_x} = some extender, E_x, on $\mathbb{E}^{M_x}$, define norm

$$\varphi_t^{\overset{\text{backgd ext}}{\overset{\shortparallel}{\text{b. ext.}}}}(x) = $$ index of backgd extender for ancestor of $\pi_x(E_x)$.

depth 2 norms.

$$\sigma = (e_0, \vec{a}, e_1)$$

ext. term — terms for generators — ext. term

defin norm $\varphi_\sigma^P(x) =$ / lin $E_0 = e_0^{M_x} \in \mathbb{E}^{M_x}$
/ lin $\vec{a}^x = \vec{a}^{M_x} \in \omega(E_0^x)^{<\omega}$
lin $E_1^x = e_1^{M_{x_1}}(\vec{a}^x) \in \mathbb{E}^{M_{x_1}}$

$$M_{x_0} = M_x$$

$$M_{x_1} = \mathrm{ult}(M_x;\ E_0^x)$$

$P_1 = \mathrm{ult}(P;\ F_0^x)$

lift / resurrect of E_0^x.

$$\begin{tikzcd}
E_1^x \quad M_{x,1} \arrow[r, "\pi_{x,1}"'] & i_{F_0^x}(N_{\alpha_x}) \quad P_1 \\
M_x \arrow[u] \arrow[r, "\pi_{x0}"'] & P_0 = P \arrow[u]
\end{tikzcd}$$

let $\varphi_\sigma(x)$ = index of background extender for ancestor of $\pi_{x,1}(E_1^x)$.

(really need for more care, but roughly these norms.)

$x \le_\sigma y \iff \exists P / \forall P$ x,y-good

$$\underbrace{\varphi_\sigma^P(x) \le \varphi_\sigma^P(y)}.$$

claim 1. is independent of P.

claim 2. gives semiscale.

Sketch. given $x_n \to x$ mod $\vec{\varphi}$. let (x_n increasing) $\in P$, x_n-good $\forall n$. let $M_\infty = \lim$. have an algorithm for lifting finite trees on M_∞ to P.