farmer 1.

Semiscales from mice.

semiscale (Π^1_3) .

start by proving ~~the~~ scale (Π^1_1) .

fix a rec. tree T on $\omega \times \omega$.

$A = \neg p[T]$.

for $x \in A$, an A-witness ~~is~~ for x

is $J_\alpha(x) \models$ " T_x is rankable." let $M_x =$
$J_\alpha(x)$ = least such for x.
[$x \in A \iff \exists$ A-witness.]

recall: scale ($\varphi_n : n<\omega$) of norms .

$\varphi_n : A \longrightarrow OR$

$$\left.\begin{array}{l} \lim_{j\to\infty} \varphi_n(x_j) = \lambda_n , \quad x_j \longrightarrow x \\ \qquad\quad \cap \\ \qquad\quad A \end{array}\right\} \; x_j \xrightarrow[\text{mod } \vec{\varphi}]{} x$$

then $x \in A$.

theory norms: for a formula ψ in the language

of x-pm ,

$$\varphi_\psi(x) = \begin{cases} 0 & \text{if } M_x \models \psi \\ 1 & \text{if } M_x \models \neg\psi . \end{cases}$$

Supp. $x_j \longrightarrow x$ mod $\vec{\varphi_\psi}$.

Let T_∞ = the limit theory

$= \{ \psi :$ eventually all m, $M_{x_m} \models \psi \}$

Let M_∞ = pointwise def., $\models T_\infty$

$\Rightarrow$ $\models V = L[\dot{x}] + \mathring{T}_{\dot{x}}$ is [illegible]

+ no proper initial segment thinks this.

to show M_∞ w.fdd :

ordinal norms : each term t

$$\varphi_t(x) = \begin{cases} t^{M_x} & \text{if } t^{M_x} \in OR^{M_x} \\ 0 & \text{o.w.} \end{cases}$$

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define $\pi : OR^{M_\infty} \longrightarrow OR$ order preserving

$$\pi(t^{M_\infty}) = \lim_{j \to \infty} t^{M_{x_j}}$$

when this is in OR^{M_∞}

generalizes M_∞ well-fdd., so $x \in A$.

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generalize. focus on Π^1_3.

$$A(x) \iff \forall y \, \exists z \, \forall w \, \psi(x, y, z, w).$$

an A - witness is 1 - small, M,

x - mouse

$M \models \exists\delta\,($ δ woodin

$\Vdash_{\mathbb{B}_\delta} \exists z\, \forall w\, \psi(\check{x}, \dot{y}, z, w)$ $\not{\&}$ +

name for $\mathbb{B}_\delta$-generic real

well-fdd. trees are rankable)

$x \in A \longleftrightarrow \exists$ an A-witness for x

(a.g. $M_1^{\#}(y)$ for all $y \in \mathbb{R}$)

for $x \in A$, M_x = least.

wants (semi) scale on A.

<u>norms</u>. use same as before. then lim $x_j \to x$ (mod $\vec{\varphi}$) and M_∞ as before.

issue: how to get M_∞ iterable?

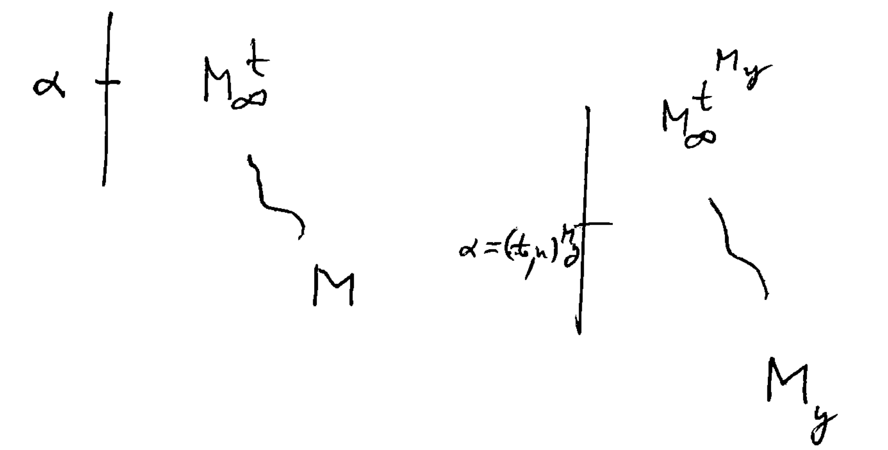
<u>finite</u> iterability. for M sound, $\rho_\omega(M) = \omega$,

finite trees ~~of~~ $\mathcal{T}$ on M coded by

finite sequences of terms t,

consider (t, u), where u is a term for

$\alpha \in OR^{M_\infty^t}$



<u>$y \in A$</u>.

defn $x \le_{(t,u)} y \iff (t,u)^{M_x} \le (t,u)^{M_y}$.

$\mathcal{T}$ length ω on M_∞

$\mathcal{T}^{M_{x_k}} \upharpoonright n$
same tree order, etc.
as $\mathcal{T} \upharpoonright n$ for suff. large k

<u>woodin:</u> proof of $\mathrm{pwo}(\Pi^1_3)$.

consider background $L[E]$-construction of A-witness.

work in V.

In $\mathbb{C}_x = L[E, x]$ constr., $= (\mathcal{W}_\alpha)_{\alpha \in OR}$

$\mathbb{C}_y = \ldots$

get α_x s.t. $\mathbb{C}_\omega(\mathcal{W}_{\alpha_x}) = M_x$

α_y — — —

set $x \leq y$ iff $\alpha_x \leq \alpha_y$.

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wan Π^1_3 - pwo. so instead:

for premice P with $x \in P$,

will defi $\mathbb{C}^P_x = (\mathcal{N}_\alpha)_{\alpha \in OR}$ (an

$L[E, x]$ constr.

<u>defi</u> $x \leq y$ iff there is $z \in \mathbb{R}$ + a

z-mouse,

$x, y \in \mathbb{R} \cap P$,

P is y-good, and

$\alpha^P_x \leq \alpha^P_y$.

<u>here</u>:

- say P is x-good iff $\exists \alpha \in OR^P$
$$\mathfrak{C}_\omega(\mathcal{N}_\alpha^{C_x^P}) = M_x .$$
then let $\alpha_x^P =$ this α.

<u>key fact</u>. $\forall z, P, z', P'$

if P is a z-mouse,

if P' is a z'-mouse,

$x, y \in P \cap P'$,

P, P' are x-good + y-good,

then $\alpha_x^P \leq \alpha_y^P \iff \alpha_x^{P'} \leq \alpha_y^{P'}$.

<u>Corollary</u>. $\leq$ is a pwo on A.

pf.: let $(x_n)_{n<\omega} \subset A$, let $z \in \mathbb{R}$,

$z \geq_T$ all x_n.

let P be a z-mouse, P x_n-good, all n.

$\Rightarrow$ $x_m \leq x_n \iff \alpha^P_{x_m} \leq \alpha^P_{x_n}$.

$\Rightarrow$ $\leq$ is a p.w.o.

ex. the p.w.o. is Π^1_3.

(hint: if P is Π^1_2 it. th, etc., but not iterable, then

$$\mathbb{R}^{M_1} \subset P \quad + \quad P | \omega_1^{M_1} = M_1 | \omega_1^{M_1}.)$$

def. work in premouse P. let $x \in \mathbb{R} \cap P$.

defi $\mathbb{C}^P_x = (N_\alpha : \alpha \in OR)$

$N_0 = J(x)$.

$N_\lambda = \liminf_{\alpha < \lambda} N_\alpha$ (lim λ)

if $N_\alpha \not\models ZF^-$, then $N_{\alpha+1} = \mathfrak{C}_\omega(N_\alpha)$

(if N_α is ω-solid)

if $N_\alpha \models ZF^-$:

case 1. $\alpha = \delta$ is a cardinal of P,

$P | \delta_{+1} \models \delta$ is woodin

let $Q \trianglelefteq P$ be $\begin{cases} P & \text{if } \delta \text{ is woodin in } P \\ Q\text{-structure} & \text{o.w.} \end{cases}$

then $\mathbb{C}^P_x \upharpoonright [\alpha, OR^Q]$

is S-construction $=$ P-construction

Q

$\gamma = \alpha$

<u>case 2.</u> there is (E, F) s.t.

- $(N_\alpha ; E)$ is a pm,
- $F \in E^P =$ extender sequence of P,
- $\nu_F = \lambda^P_\alpha$,
- $E \upharpoonright \nu_E \subset F$

then $N_{\alpha+1} = (N_\alpha ; E)$

w/ F marked index,

let $F_\alpha^{\mathbb{C}^P_x} = F$.

<u>case 3.</u> o/w

$N_{\alpha+1} = \mathcal{J}(N_\alpha)$.

pf. of key fact.

let P, Q be x,y - good. want to prove $\alpha_x^P \le \alpha_y^P$ iff $\alpha_x^Q \le \alpha_y^Q$.

plan. produce trees $\mathcal{U}, \mathcal{W}$ on P, Q

$$P \longrightarrow P'$$

$$Q \longrightarrow Q'$$

s.t. $\mathbb{C}_x^{P'} \trianglelefteq \mathbb{C}_x^{Q'}$ or $\mathbb{C}_x^{Q'} \trianglelefteq \mathbb{C}_x^{P'}$

$+$ $\mathbb{C}_y^{P'} \trianglelefteq \mathbb{C}_y^{Q'}$ or $\mathbb{C}_y^{Q'} \trianglelefteq \mathbb{C}_y^{Q'}$.

given

$$\begin{array}{ccccc} \alpha_x^{P'} \le \alpha_y^{P'} & \iff & \alpha_x^P \le \alpha_y^P \\ \shortparallel \quad \text{iff} \quad \shortparallel & & \\ \alpha_x^{Q'} \le \alpha_y^{Q'} & \iff & \alpha_x^Q \le \alpha_y^Q \end{array}$$

<u>key fact.</u> given P, Q x,y-good

$$\alpha_x^P \le \alpha_y^P \iff \alpha_x^Q \le \alpha_y^Q$$

for $\mathbb{C}_x^P$, $\mathbb{C}_y^Q$.

defi comparison (ν, w). (use padding.)

supp. then $(\nu, w) \restriction \beta+1$.

let $R = m_\beta^\nu$

$S = m_\beta^w$

for $\alpha <$ lhs. of $\mathbb{C}_x^R$

Say that R, S are (α, x) compatible iff

(1) $F_\alpha^{\mathbb{C}_x^R} = \emptyset$ iff $F_\alpha^{\mathbb{C}_x^S} = \emptyset$

(2) let $X = R \cap S$.

let $\theta = \min(\mathrm{lh}(F_\alpha^{\mathbb{C}_x^R}), \mathrm{lh}(F_\alpha^{\mathbb{C}_x^S}))$

then $F_\alpha^{\mathbb{C}_x^R} \restriction X \times \theta^{<\omega} =$

$F_\alpha^{\mathbb{C}_x^S} \restriction X \times \theta^{<\omega}$.