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J. Steel : "Distinct iteration branches"
JSL 70(4), 2005.

$M = M^{\#}_{adr}$

I

λ^M = lim of woodins + strongs up to λ^M

M active

$\rho_1(M) = \omega$, sound.

M ω_1+1 it.

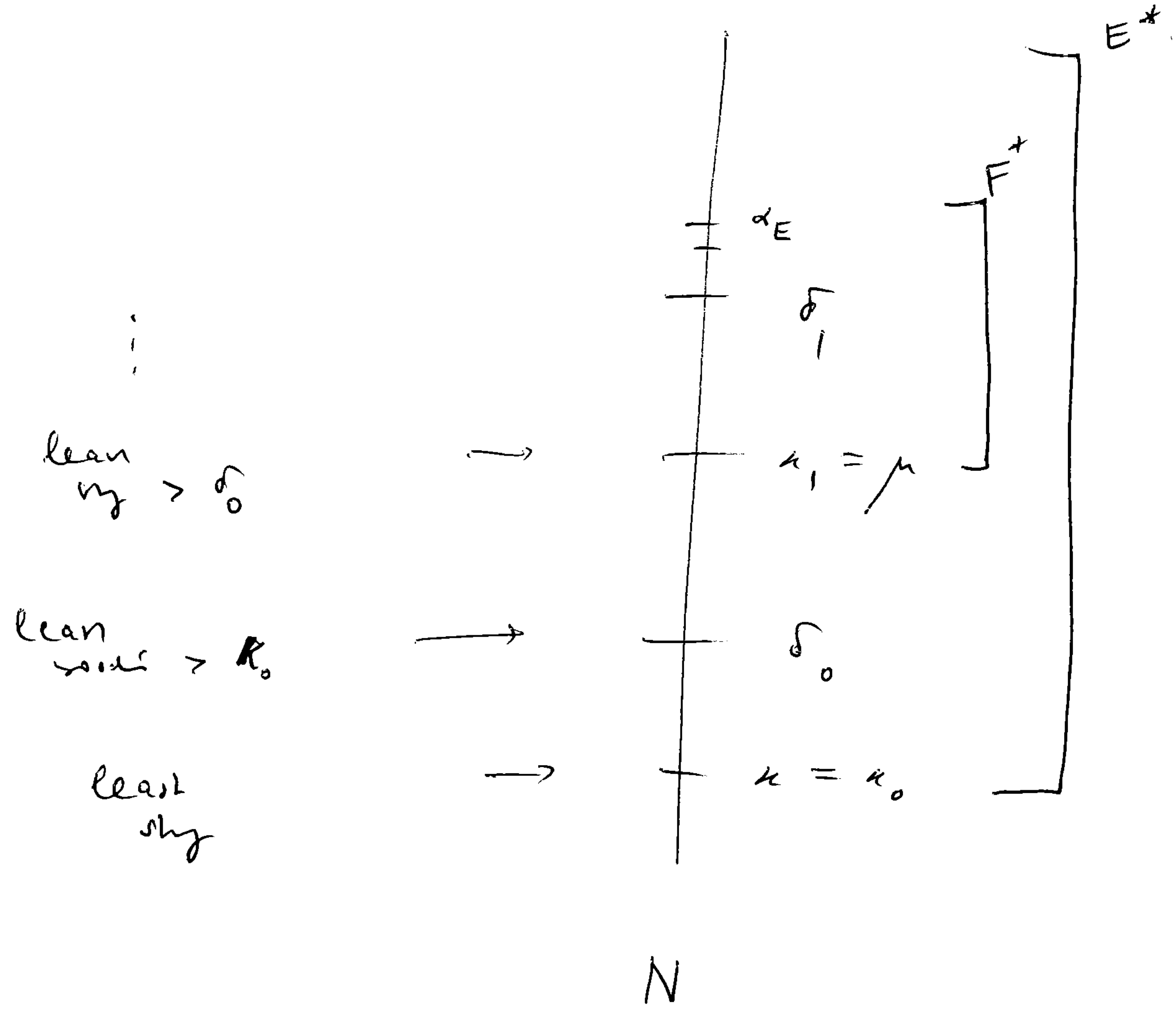
so M has a unique it. strategy, Σ.

$$\begin{array}{c} M^{\#}_{adr} \\ \parallel \\ M \xrightarrow[i]{\mathcal{U}} N \end{array}$$

g is gen. for the ext. algebra at the least woodin above the least strong,

using ext. with crit > least strong.

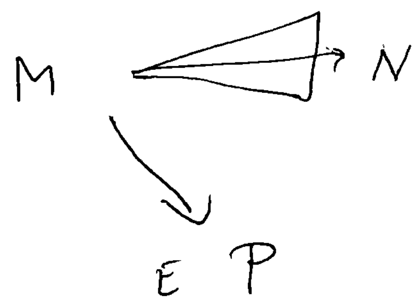
$M \in N[g]$.

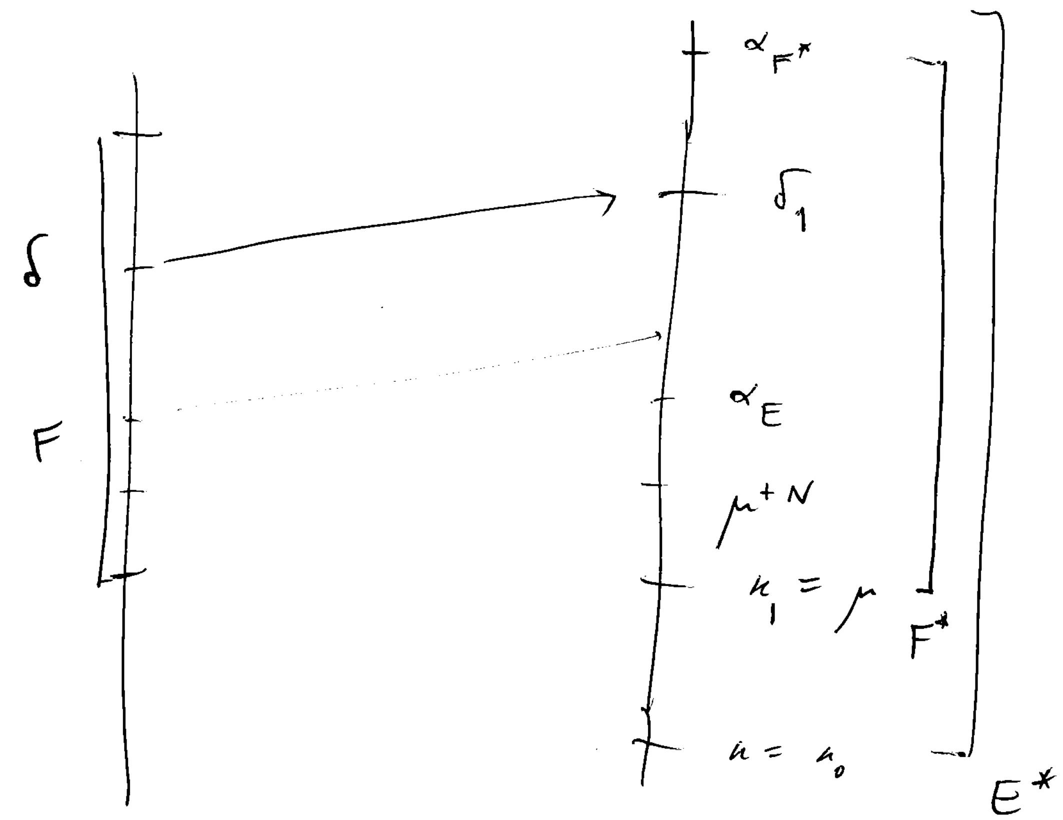


let E be a the N-[illegible].,

$$E|\mu^{+N} = E^*|\mu^{+N}.$$

say $E = E^N_{\alpha_E}$.





$Ult(N; E) \longrightarrow Ult(N; E^*)$

agree with P
up to
$i_E(\kappa)^+$

W is a genericity for absorbing E.

in ext. algebra in P at δ with crit. points $> \alpha_E$ with α_E - many generators.

b = last branch given by Σ.

$E \in Q[g, h]$.
$\uparrow$
gen. at $k(\delta)$.

claim. (a) $k(\delta) = \delta(W)$.

(b) $W \in Q[g, h]$

(c) $b \notin Q[g, h]$.

we show. the gen. iteration reaches a stage where we have (a).

as long as $i^W_{0,\eta}(\delta) > \delta(W|\eta)$, we can find a Q-structure for $W|\eta$ in $Q[g]$.

as $M, E \in Q[g, h]$,
so $P \in Q[g, h]$,

this shows $W \in Q[g, h]$,
i.e., (b),

as the Q-str. will always be an initial seg. of Q.

for (a), it suff. to prove (c).

by the picture above:

$i_E(\kappa)$ is inacc. in P,

so $k \circ i_E(\kappa)$ is inacc. in Q.

genericity then

also $i_E(\kappa) > \delta > \alpha_E$.

so $k(\alpha_E) < k \circ i_E(\delta) < k \circ i_E(\kappa)$.

but Q = hull of $k(\delta) \cup$ finite.

so it collapses $k \circ i_E(\kappa)$.

so if $b \in Q[g,h]$, then $k \circ i_E(\kappa)$

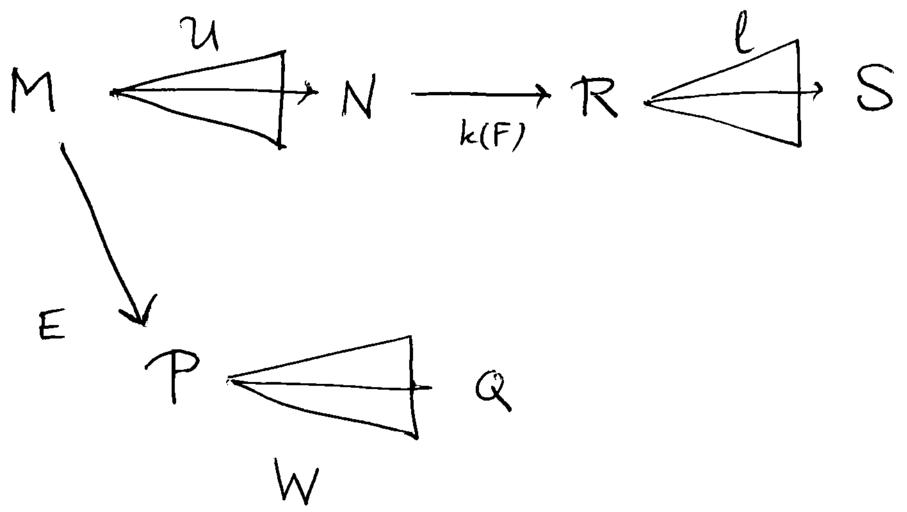
would be coll. in $Q[g,h]$.

this shows (c).

$\mathrm{crit}\,(k(F)) = \mu$.

if we want to coiterate $\mathcal{U}^\frown E^\frown W$

normally, we'd apply $k(F)$ to N :



let θ be a Woodin cardinal in R

above $lh(k(F))$.

use the ext. algebra at θ with crits

$> lh(k(F))$ to generically absorb b .

let G be gen. for ~~the ext. algebra~~ $\mathrm{Col}(\omega, \theta')$

over $S[g, h]$ (S = last model of gen. iteration) .

$\theta' = l(\theta)$

l = map from $R \longrightarrow S$.

so have

$$b \in S[g, h, G].$$

point: $b \notin OD^{S[g,h,G]}_{S[g,h]}$.

o.w. $b \in S[g,h]$.

this would imply $b \in Q[g,h]$. $\lightning$

~~we show~~

~~in~~

<u>note</u>: if $x \in N|\lambda^N[g]$ then

$$M^{\#}_{adr}(x) \in N[g].$$

why? $M \in N[g]$. if α is an active level of N s.t. $x \in N\|\alpha[g]$ and $\mathrm{crit}(E^N_\alpha) = \underset{\kappa}{\overset{\kappa_0}{\shortparallel}}$ = the least msb of N, M, then $x \in \mathrm{Ult}(M; E_\alpha)[g]$.

then perform a local $L[E]$ construction in $\mathrm{ult}(M, E_\alpha)[g]$ over x.

the same arg applies to S.

we show: in $S[g, h, G]$ there is a map

$$(*) \quad \pi : \underset{\underset{\textstyle Q}{\shortparallel}}{M_b^W} \longrightarrow M^{\#}_{adr}(S_0 \mid \tau),$$

$\tau =$ the least cardinal of S_0 strong to κ^{S_0}.

$S_0 =$ the local $L[E]$ constr.
run inside $S[g, h, G]$
using extenders ~~to~~ with crits
high up.

there must then be some c s.t. $(*)$
holds with c in place of b as
o.w. b would be $OD^{S[g, h, G]}_{S[g, h]}$.

$$\begin{tikzcd}
 & & & j^*(S_0^+) \\
M \arrow[r] & M_b^W \arrow[r, "i"] & \mathcal{Z} \arrow[ur, "\psi"] \arrow[r, dash, dashed] & Y \\
 & & S_0^+ \arrow[u, "j"] \arrow[r, dash, dotted] & S \arrow[u, "j^*"']
\end{tikzcd}$$

$$M^{\#}_{adr}(j^*(S_0|\tau)) = Hull_{\Sigma_1}^{ult(j(S_0),\,u)}(j^*(\tau)+1)$$

U = the measure of order 0 on crit pt $j^*(\tau)$

hence
$$\begin{tikzcd}
M_b^W \arrow[r, "\varphi"] & ult(S_0^+,\, u) \\
 & M^{\#}_{adr}(j(S_0|\tau)) \arrow[u, "\psi"]
\end{tikzcd}$$

so $ran(\varphi) \subset ran(\psi)$.