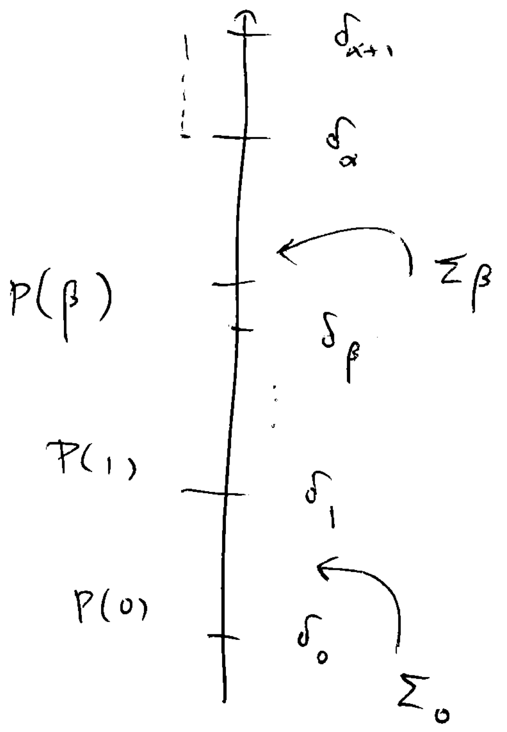
Nam Trang 2

Lsa - small hod pm



β limit

$P(\beta) = Lp^{\oplus \Sigma}_{\gamma<\beta}\,\gamma(P|\delta_\beta)$.

$\|$

$P|\delta_\beta^+$,

both computed in P.

δ_α lim of woodins

δ_α is strong to $\delta_{\alpha+1}$

$\delta_{\alpha+1}$ is the largest woodin

$\Big\}$ lsa

lsa - small : no strict initial segment of P is an lsa hod pm.

for P of lsa-type :

$P^b = Lp^{\oplus \Sigma}_{\gamma<\alpha}\,\gamma(P|\delta_\alpha)^P$, δ_α 2nd largest wood'

$Lp^{\oplus \Sigma_\gamma}_{rep}(P \mid \delta_\beta)$: no extender w/ crit δ_β.

Suppose (P, Σ) is an lsa-small hod pair s.t. Σ has branch condensation and is Γ-fullness preserving for some pointclass Γ.

<u>defin</u>. Σ is strong Γ-fullness preserving iff f.a. $(\vec{\mathcal{I}}, S) \in I(P, \Sigma)$, i.e. S is a Σ-it. of P via $\vec{\mathcal{I}}$, s.t.

either
a) $\pi^{\vec{\mathcal{I}}, b} : P^b \longrightarrow S^b$ exists iff P is of lsa-type, or
b) $\pi^{\vec{\mathcal{I}}} : P \longrightarrow S$ exists,

then letting $\pi^* = \pi^{\vec{\mathcal{I}}, b}$ or $\pi^{\vec{\mathcal{I}}}$, resp.,

if $\tau : R \longrightarrow S' \trianglelefteq S$, τ Σ_1, $crit(\tau) \geq S^b$, then

$\Sigma^{\tau}_{\vec{\mathcal{I}}, S}$ is Γ-fullness preserving.

(and s.th. analogous in case b))

def. Σ has strong condensation iff Σ has branch condensation + whenever $P^b \lhd Q \unlhd P$ and

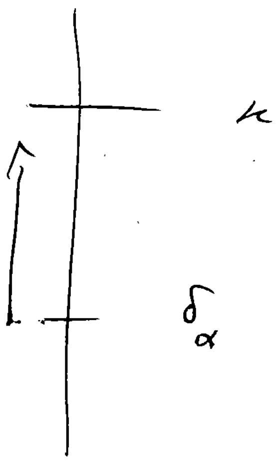
$$Q \xrightarrow[\Sigma]{\vec{\mathcal{I}}} S \quad \text{s.t.} \quad \pi^{\vec{\mathcal{I}},b} : Q^b \longrightarrow S^b$$

exists and $\sigma : Q \longrightarrow_{\Sigma_0^{(n)}} R$, $S^b \lhd R \unlhd S$, ~~...~~ $\sigma \supset \pi^{\vec{\mathcal{I}},b}$, n s.t. $\rho_{n+1}(Q) < \mathrm{crit}(\sigma) \le \rho_n(Q)$,

then $\Sigma^{\sigma}_{R,\vec{\mathcal{I}}} = \Sigma_Q$.

here, P has lsa-type or is of limit type.

limit type:



δ_α strong up to κ, but κ need not be a Woodin.

<u>Th.</u> Supp. (P, Σ) is an lsa-small hod pair s.t. Σ has strong condensation and is strongly Γ-fullness preserving for some inductive-like $\Gamma \models$ "AD^+ + $\underbrace{\text{strong mouse capturing}}$"

then $\forall \kappa \in \mathrm{card}(P)$ $\quad P \models \square_\kappa$.

~~notation~~ <u>notation :</u>

fix κ.

assume $\kappa \in [\delta_\alpha^+, \delta_{\alpha+1})$.

$S = \{ \kappa < \tau < \kappa^+ : \ P|\tau \prec_{\Sigma_\omega} P|\kappa^+,$

$P|\tau$ is passive $\}$

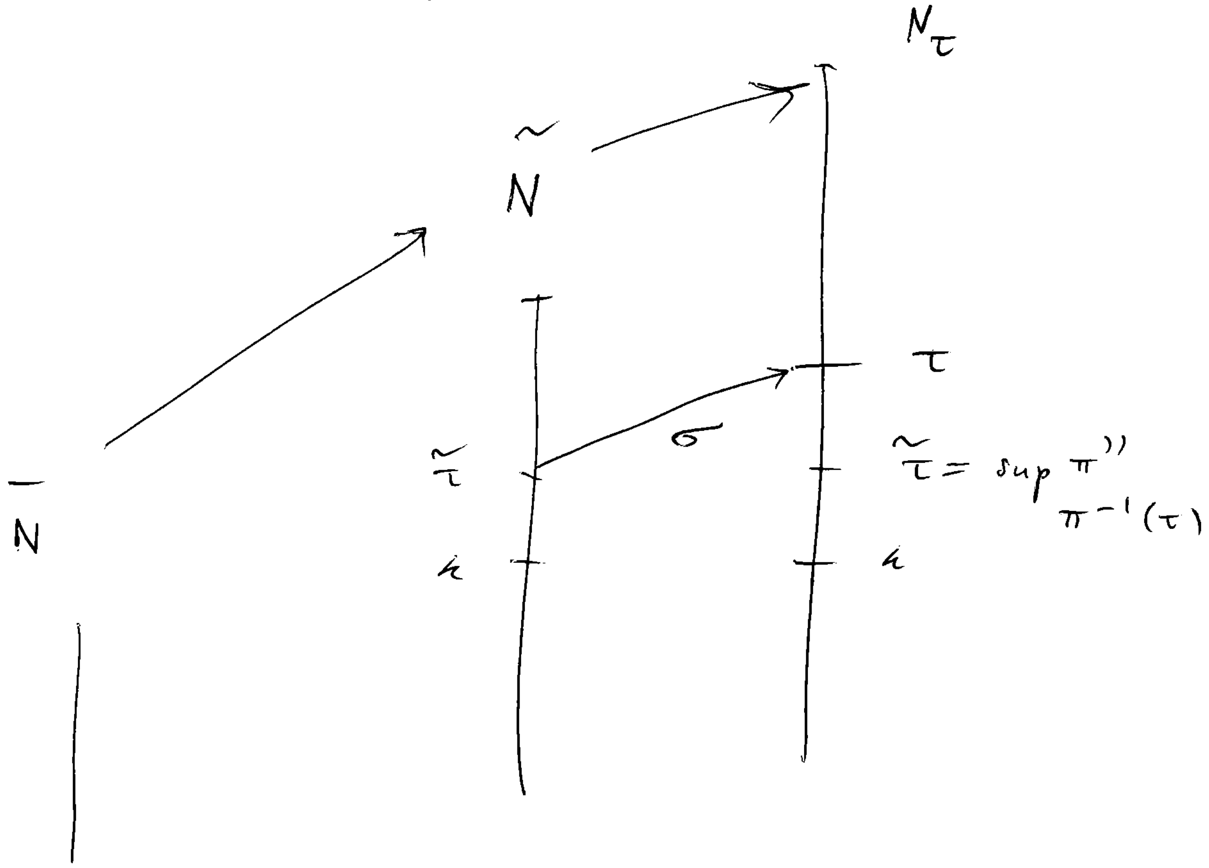
$\forall \tau \in S$ have to define

"the can. coll. structure" N_τ

for τ.

Say you pick $N_\tau \trianglelefteq P$,

$\tau = \kappa^{+N_\tau}$, $\rho_w(N_\tau) = \kappa$.



w.t.s: $\tilde{N} = N_{\tilde{\tau}}$.

if $E^{top}_{N_\tau} \neq \emptyset$, $\rho_1(N_\tau) = \kappa$,

$crit(E^{top}_{N_\tau}) < \kappa$,

then $\tilde{N}$ will be a protomouse.

if $crit(E^{top}_{N_\tau}) > \delta_\alpha$, can ~~do~~ imitate

the L[E]-solution.

Say $crit(E^{top}_{N_\tau}) = \delta_\alpha$.

in $F = E^{\tilde N}_{top}$

look at $ult(\tilde N | \nu, F)$, w

$\nu < \delta_\alpha^+$ is largest s.t. F measures

$\mathcal{P}(\delta_\alpha) \cap \tilde N | \nu$; $\rho_w(\tilde N | \nu) = \delta_\alpha$.

~~~~$\tilde\tau$~~

$ult(\tilde N | \nu, F)$ is a candidate for

$N_{\tilde\tau}$, but $N_{\tilde\tau} \not\triangleleft \mathcal{P}$,

as the new woodins aren't δ_β's.

<u>defn.</u> (divisor) fix $N_\xi^{\triangleleft \mathcal{P}}$ as before.

Supp. $\rho_{n+1}(N_\xi) \le k < \rho_n(N_\xi)$.

we say (μ, q) is a <u>divisor</u> if

$\exists \lambda$ s.t. letting $r = p_{N_\xi} \setminus q$,

(a) $\mu \le k < \lambda < \rho_n(N_\xi)$,
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(b) $q = p_{N_\xi} \cap \lambda$

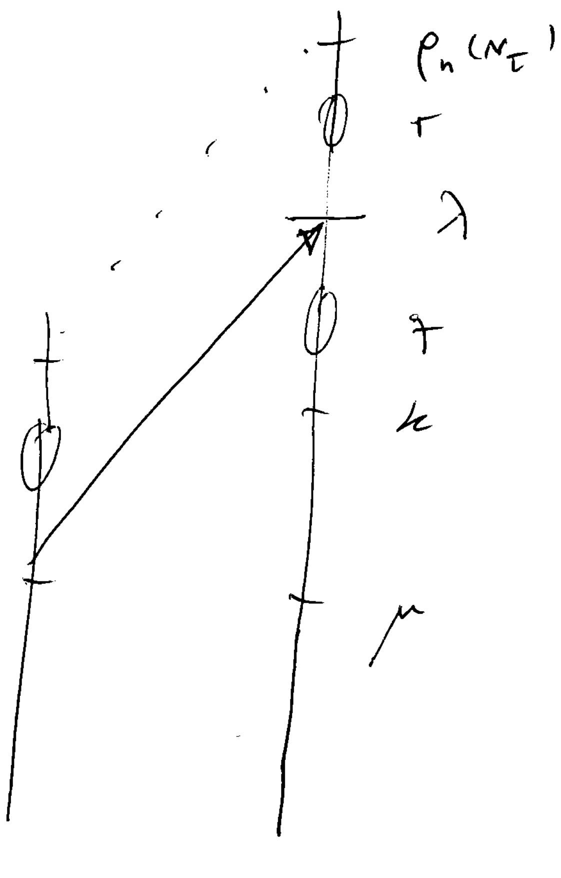
(c) $\tilde{h}^{n+1}_{N_\xi}(\mu \cup \{\tau\}) \cap \rho_n(N_\xi)$ is cofinal in $\rho_n(N_\xi)$,

(d) $\lambda = \min((\tilde{h}^{n+1}_{N_\xi}(\mu \cup \{\tau\}) \cap \mathrm{OR}) \setminus \mu)$

Say (μ, q) is strong iff (μ, q) is a divisor and

$\forall \xi < \mu \ \forall x = \tilde{h}^{n+1}_{N_\xi}(\xi, p_{N_\xi})$

$x \cap \mu \in \tilde{h}^{n+1}_{N}(\mu \cup \{\tau\})$



the canonical strong divisor for N is the strong divisor (μ_N, q_N) with the following properties.

- q_N is shortest among q s.t.

  $\exists \mu$ (μ, q) is a strg divisor.

- μ_N is maximal w.r.t. strg divisors of the form (μ, q_N).

<u>defn</u>. (N^*_τ).

~~for $\tau \in S$ s.t. E^{top}~~

let $N^*_\tau \trianglelefteq P$ be the corr. level of P for τ.

<u>def</u>. (N_τ).

Supp. $\tau \in S$ is such that

$\mathrm{crit}(E^{top}_{N^*_\tau}) = \delta_\alpha$, $\rho_1(N^*_\tau) = \kappa$,

and Supp. there is a pointclass

$\Omega \subsetneq \Gamma$ s.t. there is a hod pm R which is a coll. str. for τ,

$\gamma_\tau = o^{N^*_\tau}(\delta_\alpha)$, a cutpoint of R

mitchell ord

$P|\gamma_\tau \lhd R$

R has infinitely many woodin cardinals, say with sup $= \lambda > \gamma_\tau$

+ the can. divisor of R exists and is of the form (δ_α, q) and $\lambda = \lambda(\delta_\alpha, q)$.

+ R generates Ω.

if (Ω, R) exists, then let N_τ be the generator for the min. such Ω.

if (Ω, R) doesn't ex., then $N_\tau = N_\tau^*$.

for other $\tau \in S$, also let $N_\tau = N_\tau^*$.