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Motivation: Want to compare hod pairs (P, Σ), (Q, Λ)

problem: copying Σ, Λ.

want to reach $i: P \xrightarrow{\mathcal{T}} P^*$ via Σ

$j: Q \xrightarrow{\mathcal{U}} Q^*$ via Λ

s.t. $P^* \trianglelefteq Q^*$ or $Q^* \trianglelefteq P^*$ and

the tails $\Sigma_{\mathcal{T}, P^*}$, $\Lambda_{\mathcal{U}, Q^*}$ are compatible.

Convention. Our strategies act on stacks $\vec{\mathcal{T}}$ of normal trees: $\mathcal{T}_0$ on base model, $\mathcal{T}_{\alpha+1}$ on last model of $\mathcal{T}_\alpha$, $\mathcal{T}_\lambda$ on dir. lim of $\mathcal{T}_\alpha$, $\alpha < \lambda$.

normal: lengths of extenders increasing + if E is used before F on the same branch, then $\text{crit}(F) \geq \lambda_E = i_E(\text{crit}(E))$

(or $\nu(E) \leq \text{crit}(F)$)

$$\Sigma_{\vec{I}}(\vec{u}) = \Sigma(\vec{I}^\frown \vec{u}) .$$

proof : iterating away strategy disagreements ...

how to do it ?

$$I \} \qquad\qquad \{ u$$

$$P_\alpha \qquad Q_\alpha \qquad \text{say } \Sigma_{I,R} \neq \Sigma_{u,R}$$

idea : "use branch embedding as extender"

problem : <u>many generators</u> !!

if there are no ext. overlapping woodins in the hod mice, then there are ways out by woodin, Sargsyan.

but not in general.

<u>potential solution</u> : compare (P, Σ) and (Q, Λ) with a common background construction.

(*) (P, Σ) : for any suff. good background universe <u>capturing</u> (P, Σ), (P, Σ) iterates into its $L[\vec{E}]$ or its hod mouse constr., no strategy disagreement, no extender disagreement.

here: "capturing" means the following.

let $\mathcal{B}$ = background universe

won: $P \in HC^{\mathcal{B}}$, Σ has u.b. code in $\mathcal{B}$.

this proof needs condensation properties for Σ. (to construct Σ with these properties we need some form of UBH in the background universe when we constructed ~~it~~ (P, Σ).)

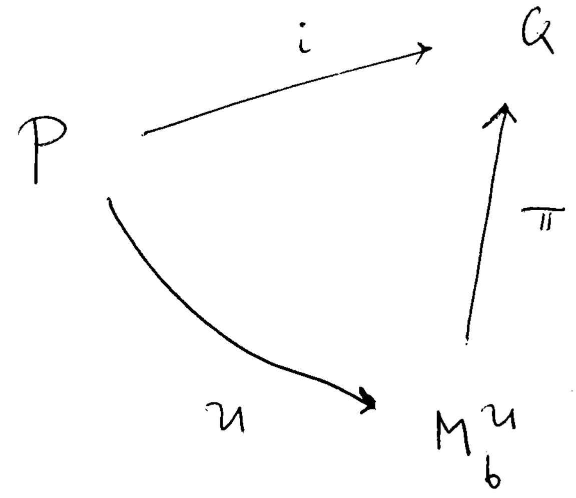
<u>Sample condensation properties:</u>

(1) hull condensation.

if $\mathcal{T}$ is by Σ and $\mathcal{U}$ is a hull of $\mathcal{T}$, then $\mathcal{U}$ is by Σ.

Sargsyan showed UBH yields such (P, Σ).

(2) branch condensation :



i by Σ,
$\pi \circ i_b^{\mathcal{U}} = i$
$\mathcal{U}$ by Σ,
then
$\Sigma(\mathcal{U}) = b$.

e.g. M is a tame mouse, $\rho_\omega(M) = \omega$,
M sound, M iterable.

then M has a unique Σ,

$\Sigma(\mathcal{U})$ = unique b s.t. $M_b^{\mathcal{T}}$ is iterable.

outline of $(*)(P, \Sigma)$, when Σ has branch condensation.

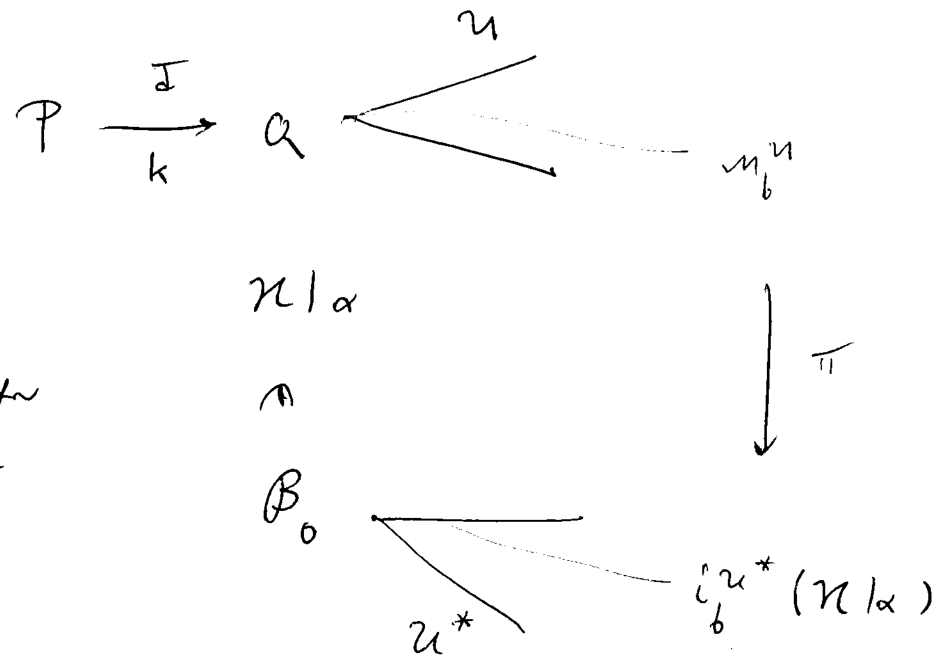
$\mathcal{N}$ some level of the $L[\vec{E}]$ (or hod mouse).

(P, Σ), $P \in \mathcal{N}C$, Σ u.b.

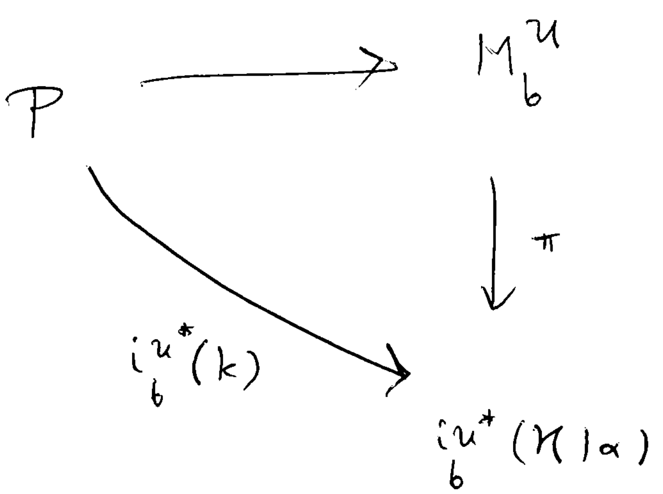
$$P \longrightarrow Q$$

say $Q|\alpha = \mathcal{N}|\alpha$, $\Sigma_{\mathcal{T}, Q|\alpha}(\mathcal{U}) \neq \Omega(\mathcal{U})$, where Ω is the strategy for $\mathcal{N}$.

$\mathcal{B}_0$ = background for the construction of $\mathcal{N}$



$i_b^{\mathcal{U}^*}(\mathcal{T})$ has last model $i_b^{\mathcal{U}^*}(\mathcal{N}|\alpha)$, and $i_b^{\mathcal{U}^*}(\mathcal{T})$ is by Σ (because Σ is universally Baire)



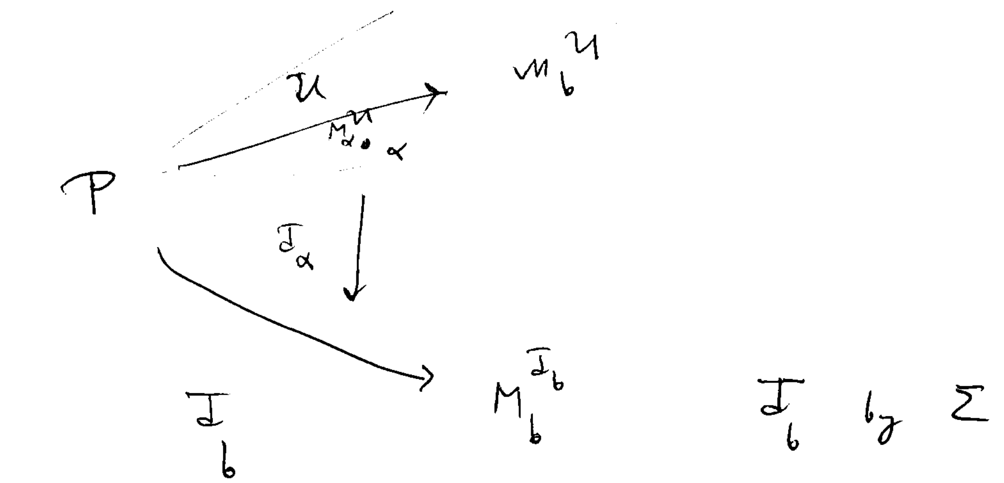
hence b is acc. to Σ, so $b = \Sigma_{\mathcal{T}}(\mathcal{U})$. $\dashv$

problem: branch condensation probably fails once we get to extenders overlapping woodins.

($\Sigma(\mathcal{J}) \not\ni$ the unique c s.t. $M_c^{\mathcal{J}}$ is iterate.

in gen. $\Sigma(\mathcal{J}) =$ unique b s.t. $\Phi(\mathcal{J}^\frown b)$ is iterate.

<u>phalanx condensation.</u>



possibly, background conditions yield (P, Σ) with phalanx condensation.

can show $(*)(P, \Sigma)$ if

Σ has phalanx condensation.

another condensation property:

normalizing well: given $(\mathcal{T}, \mathcal{U})$, a

2-stack of normal trees,

can construct a normal tree $W(\mathcal{T}, \mathcal{U})$,

minimal in some sense, s.t. every

$$M_\alpha^{\mathcal{U}} \xrightarrow[\pi_\alpha]{} M_{\varphi(\alpha)}^{W(\mathcal{T}, \mathcal{U})}.$$

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let $W(\mathcal{T}, \mathcal{U}^\frown b) = W_b$.

in our branch condensation arg,

W_b is a pseudo hull of $i_b^{\mathcal{U}*}(\mathcal{T})$.

our Σ will have strong hull condensation

which means W_b is by Σ.

<u>defn</u>. Σ 2-<u>normalizes well</u> iff whenever $(\mathcal{I}, u)$ is by Σ, then $W(\mathcal{I}, u)$ is by Σ

<u>so</u> if $c = \Sigma_{\mathcal{I}}(u) = \Sigma(\mathcal{I}^\frown u)$, then W_c is by Σ.

can show that this implies [illegible]

<u>defn</u>. $W(\mathcal{I}, u)$

normalizing well

very full condensation

background construction

lifting iterability

fine structure : do it for pure ext. premice
λ – indexing .

<u>convention.</u> every premouse $\mathcal{M}$ has a
associated $k(\mathcal{M})$ (its degree of
soundness)

$ult(M;E) = ult_{k(M)}(M;E)$

gives rise to $r\Sigma_{k+1}$ el. embedding .

$\rho(M) = \rho_{k(M)+1}(M)$.

let $\mathcal{T}$ be a normal tree on M,
last model Q .
let F be on the Q-sequence .

define $W(\mathcal{T}, F)$

let $\alpha = \alpha^{\mathcal{T},F}$ be least s.t.
F is on the $M_\alpha^{\mathcal{T}}$ - sequence ,

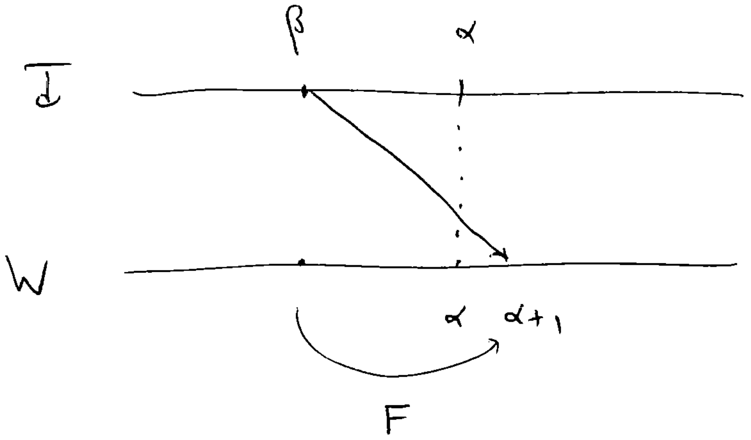
let $\beta = \beta^{\mathcal{T},F}$ = least β s.t.

$$\mu = crit(F) < \lambda_{E_\beta^{\mathcal{T}}} .$$

$$W(\mathcal{T}, F)\restriction(\alpha^{\mathcal{T},F}+1) = \mathcal{T}\restriction(\alpha^{\mathcal{T},F}+1)$$

$$M^W_{\alpha+1} = \mathrm{ult}(P, F)$$

$P \trianglelefteq \mathcal{M}^{\mathcal{T}}_\beta$, dictated by normality.



remaining part of W given by copying.

defin $M^W_{\varphi(\gamma)}$, $\pi^{\mathcal{T},F}_\gamma : \mathcal{M}^{\mathcal{T}}_\gamma \longrightarrow M^W_{\varphi(\gamma)}$

$$\begin{array}{ccc} M_\gamma & \xrightarrow{E^{\mathcal{T}}_\xi} & M_{\xi+1} \\ \downarrow \pi^{\mathcal{T},F}_\gamma & & \downarrow \pi_{\xi+1} \\ M_{\varphi(\gamma)} & \xrightarrow{\pi_\xi(E^{\mathcal{T}}_\xi)} & M_{\varphi(\xi+1)} \end{array}$$

$\gamma = T\text{-pred}(\xi+1)$

for crit $(E_\xi^{\bar\alpha}) < \lambda_\beta^{\bar\alpha} = \sup\{\lambda_{E_\alpha^{\bar\alpha}} : \alpha < \beta\}$

or

crit $(E_\xi^{\bar\alpha}) \geq \mu$

o.w. $\beta = T\text{-pred}(\xi+1)$

$$\begin{array}{ccc} M_\beta & \xrightarrow{E_\xi} & M_{\xi+1} \\ \downarrow \text{id} & & \downarrow \pi_{\xi+1} \\ M_\beta & \xrightarrow[\pi_\xi(E_\xi)]{} & M^W_{\varphi(\xi+1)} \end{array}$$

$$\varphi(\gamma) = \begin{cases} \gamma & \text{if } \gamma < \beta \\ (\alpha+1) + (\gamma - \beta) & \text{if } \gamma \geq \beta \end{cases}$$

can write

$$W(\mathcal{I}, F) = \mathcal{I}^{< lh(F)} {}^\frown \langle F \rangle {}^\frown i_F{}'' \mathcal{I}^{> crit(F)}$$

branch extender

$\mathcal{T}$ normal on P

$\mathcal{U}$ on last model of $\mathcal{T}$.

$W(\mathcal{T}, \mathcal{U})$.

notice F could have come from

M^S_α, where S is a normal tree on P.

say $\beta = \beta^{S,F}$, $\alpha = \alpha^{S,F}$.

as long as $\mathcal{T}$ on P is normal we

$$M^{\mathcal{T}}_\beta = M^S_\beta.$$

(so in fact

$\mathcal{T} \restriction \beta+1 = S \restriction \beta+1$.

true if $\mathcal{T}, S$ by same

strategy)

can make sense of $W(\mathcal{T}, F)$.

$$W(\mathcal{T}, F) = S \restriction \alpha+1 {}^\frown \langle F \rangle {}^\frown i_F \text{''} \mathcal{T} > \text{crit}(F)$$

now define $W(\mathcal{T}, \mathcal{U})$ by ind. on γ

we'll define $W_\gamma = W(\mathcal{T}, \mathcal{U} \restriction \gamma+1)$.

$W_0 = \mathcal{J}$.

similarly, define, for $\nu <_u \gamma$

$\varphi_{\nu,\gamma} : \text{lh}\, W_\nu \longrightarrow \text{lh}\, W_\gamma$

(possibly partial, happens iff $(\nu, \gamma]_u$ has a drop) .

have $\pi^{\nu,\gamma}_\tau : M^{W_\nu}_\tau \longrightarrow M^{W_\gamma}_{\varphi_{\nu,\gamma}(\tau)}$

for $\tau \in \text{dom}\, \varphi_{\nu,\gamma}(\tau)$.

these have various agreement properties .

$\varphi_{\nu,\gamma}$, $(\pi^{\nu,\gamma}_\tau : \tau \in \text{dom})$ is an

example of a pseudo hull

have also : let R_γ = last model of W_γ

have $\sigma_\gamma : M^{\mathcal{U}}_\gamma \longrightarrow R_\gamma$

set $F_\gamma = \sigma_\gamma (E^u_\gamma)$.

$W_{\gamma+1} = W (W_\nu , F_\gamma)$,

where $\nu = u -$ pred $(\gamma+1)$.

β^{W_ν, F_γ}

$\alpha_\gamma =$ least α s.t. F_γ is on the $M^{W_\gamma}_\alpha$ - seq .

so $W_{\gamma+1} \restriction (\alpha_\gamma + 1) = W_\gamma \restriction (\alpha_\gamma + 1)$.

$$\varphi_{\nu, \gamma+1} (\xi) = \begin{cases} \xi & \text{if } \xi < \beta^{W_\nu, F_\gamma} \\ (\alpha_\gamma + 1) + (\xi - \beta) & \text{if } \xi \geq \beta \end{cases}$$

defined this way if $\nu \mapsto \gamma + 1$ does not drop in model or degree .

o.w. $\operatorname{dom} \varphi_{\nu,\gamma+1} = \beta+1$.

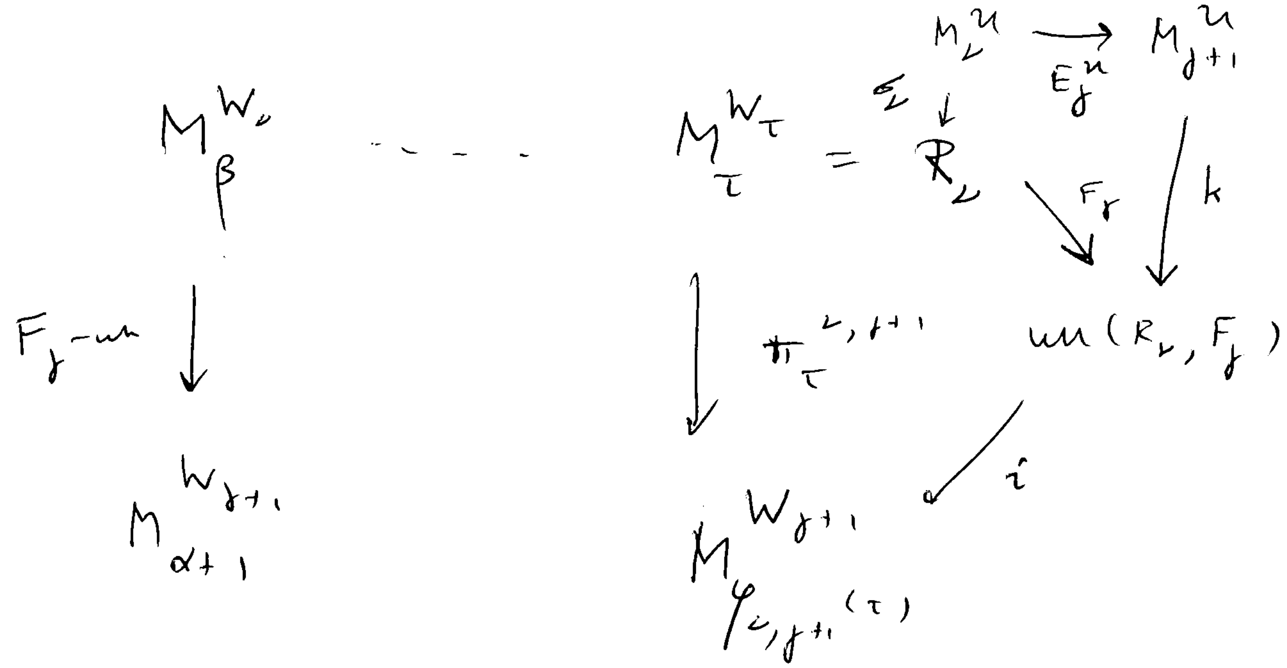
$$\pi_\tau^{\nu,\gamma+1} : M_\tau^{W_\nu} \longrightarrow M_{\varphi_{\nu,\gamma+1}(\tau)}^{W_{\gamma+1}}$$

$\varphi_{\eta,\gamma+1}$ for $\eta <_u \nu$

$\| $

$\varphi_{\nu,\gamma+1} \circ \varphi_{\eta,\gamma}$

$$\pi_\tau^{\eta,\gamma+1} = \pi_{\varphi_{\eta,\nu}(\tau)}^{\nu,\gamma+1} \circ \pi_\tau^{\eta,\nu}.$$



$$\sigma_\nu \restriction lh\, E_\nu^u = \sigma_\gamma \restriction lh\, E_\nu^u$$

$$\sigma_{\gamma+1} = i \circ k .$$