Farmer Schlutzenberg

Thm.
Let M be k-sound
(k, ω_1+1) it. the pm = premouse.
then

- M is $(k+1)$-solid, $(k+1)$ universal
- if $k \geq 1$, then ~~M~~ M is dodd-~~[illegible]~~ sound
- $(k+1)$ condensation holds

classical proofs require
$(k, \omega_1, \omega_1+1)$ iterability.

quest: does normal iterability imply full iterability?

main difference: replace phalanxes by bicephali.

<u>illustration</u> (condensation)

supp. M k-sound, (k, ω_1+1) iterable.

$\pi : H \longrightarrow M$ near k-embedding.

H is $(k+1)$-sound

$\mathrm{crit}(\pi) \geq \rho$

ρ is an M-cardinal, $\rho_{h+1}^H = \rho < \rho_h^H$

π $r\Sigma_{h+1}$-elementary.

$M \models$ "condensation holds for proper segments"

ρ is regular in H

$(\rho^+)^H < (\rho^+)^M$

$M|\rho$ is passive

$M|\rho^{+M}$ not active type 3

case assumptions

<u>Want:</u> $H \lhd M$.

<u>define</u> a <u>bicephalus</u> is a structure

$$\mathcal{B} = (H, J, \rho, \gamma_0, \dots, \gamma_{n-1})$$

- H, J are premice
- ρ is a cardinal in both H, J
- $H \| \rho^{+H} = J \| \rho^{+H}$

  $\Longrightarrow \rho^{+H} \le \rho^{+J}$.

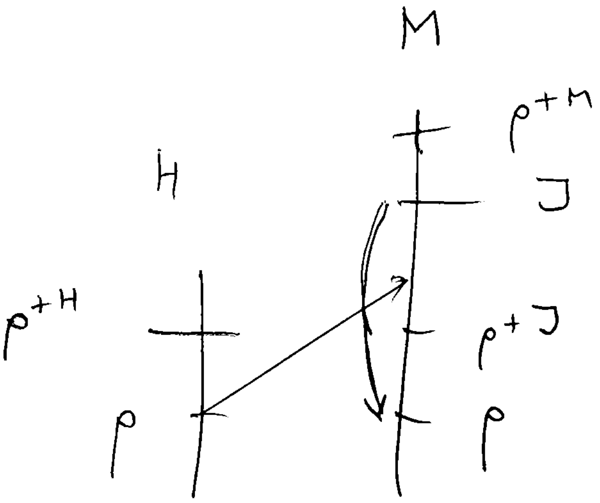
$\|$ passive
$|$ active, potentially

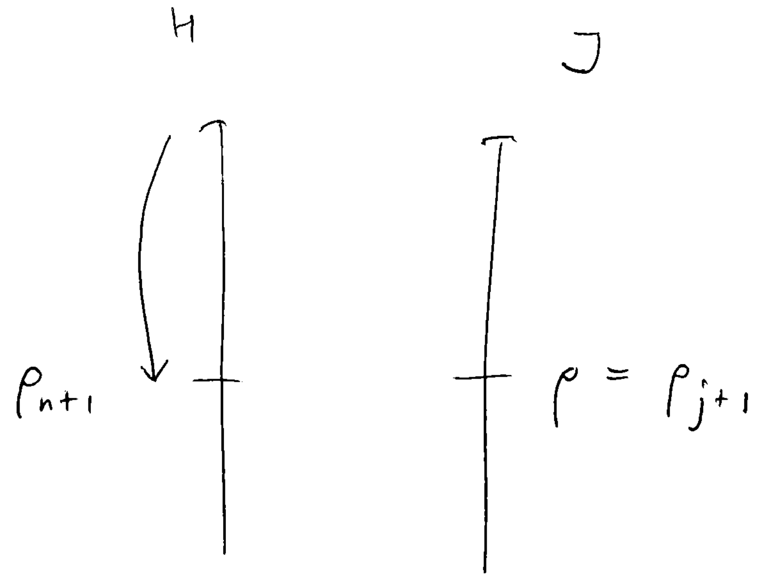
Scenario: might want to prove $H \trianglelefteq J$.

back to $\pi : H \longrightarrow M$.

let $J \triangleleft M$ s.t. $\rho_\omega^J = \rho$,

$\mathcal{P}(\rho)^J = \mathcal{P}(\rho)^H$.





let $B = (H, J, \rho)$

supp. $H \neq J$.

plan: compare B with itself

prove comparison terminates

prove comparison does not terminate.

issues:
- ultrapowers of B
- preservation of fine structure
- iteration trees
- iterability
- rules of comparison
- analyze comparison

E is semi-close to a pm N iff
$N | (\kappa_E^+)^N = \mathrm{ult}(N; E) | \kappa_E^+$

<u>ultrapowers</u> let E be an extender which
is semi-close to ~~both~~ B, i.e.
to both models of B, and
$\kappa_E < \rho$.

$$\begin{array}{c} H' \\ \shortparallel \end{array} \quad \begin{array}{c} J' \\ \shortparallel \end{array} \quad \begin{array}{c} \rho' \\ \shortparallel \end{array}$$

then $\mathrm{ult}(B, E) = (\mathrm{ult}_h(H; E), \mathrm{ult}_j(J, E), i_E^H(\rho))$

because $\rho^{+H} \leq \rho_h^H$,
$i_{E,h}^H \upharpoonright \rho^{+H} = i_{E,0}^N \upharpoonright \rho^{+H} = i_{E,j}^J \upharpoonright \rho^{+J}$
$\leadsto$ $N = H \| \rho^{+H}$
and embeddings are continuous at ρ^+.

<u>defn</u>. let $C = (M, N, \lambda)$ be a
bicephalus.

C is <u>exact</u> iff $M \| \lambda^{+M} = N \| \lambda^{+N}$.
C is <u>nontrivial</u> iff $M \not\trianglelefteq N$

C is projecting iff $\exists\, m, n$

$$\rho^M_{m+1} \le \lambda < \rho^M_m, \quad M \quad m\text{-sound}$$

$$\rho^N_{n+1} \le \lambda < \rho^N_n, \quad N \quad n\text{-sound}$$

call $(m,n) =$ the degree of $C = \deg(C)$

if projecting, C is sound iff

M is $(m+1)$-sound and

N is $(n+1)$-sound.

claim. B' has all four properties

and $\deg(B') = \deg(B) = (h, j)$,

$$\rho^{H'}_{h+1} = \rho' = \rho^{J'}_{j+1}.$$

proof that B' is non-trivial.

supp. $H' = J' \Rightarrow h = j$.

$\Rightarrow \exists\, r\Sigma_{h+1}$ fmla φ, $\alpha < \rho$

$H \neq J$ gris

$H \models \varphi(\alpha, p_{n+1}^{H}) \iff$

$J \models \neg \varphi(\alpha, p_{n+1}^{J})$.

but then

$H' \models \varphi(\alpha', p_{n+1}^{H'}) \iff$

$J' \models \varphi(\alpha', p_{n+1}^{J'})$. $\not\downarrow$

know $p_{n+1}^{H} \mapsto p_{n+1}^{H'}$ by ind. hypo.

$p_{n+1}^{J} \mapsto p_{n+1}^{J'}$

iteration trees.

similar to a ~~k~~ k-max. tree.

tree $\mathcal{I} = (<_{\mathcal{I}}, (E_\alpha)_{\alpha+1 < \mathrm{lh}(\mathcal{I})},)$

associated structures

- $(B_\alpha, H_\alpha, J_\alpha)_{\alpha < \lambda}, (B^*_{\alpha+1})_{\alpha+1 < \lambda}$

- sets $\mathcal{D}, \mathcal{B}, \mathcal{H}, \mathcal{J} \subset \lambda$.

degree function $\longrightarrow \omega^2 \cup \omega$

embeddings $i_{\alpha,\beta}$, $j_{\alpha,\beta}$.

ordinals ν_α , ρ_α .

$\mathcal{D}$ = ~~drops~~ drops

B_α is a tricephalus iff $\alpha \in \mathcal{B}$

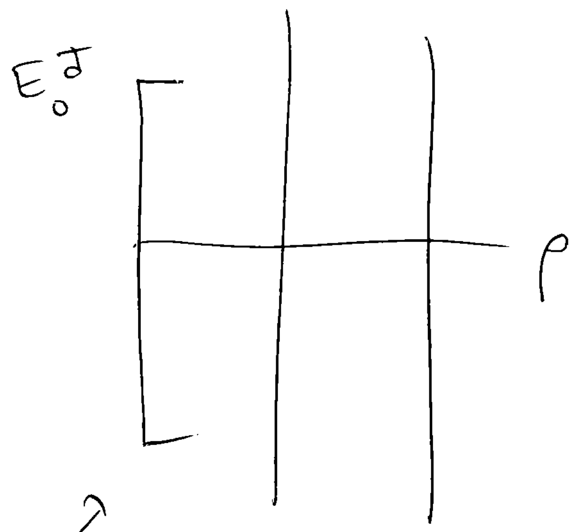
$\Downarrow$

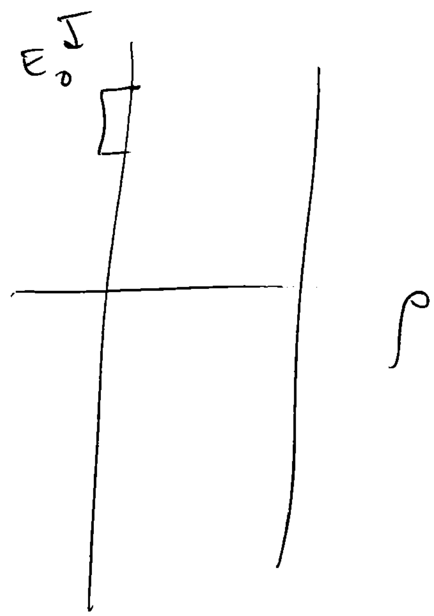
$$B_\alpha = (H_\alpha, \mathcal{J}_\alpha)$$

first ext.

$$B = (H, \mathcal{J})$$

apply it to H





apply it to B

iterability proof:

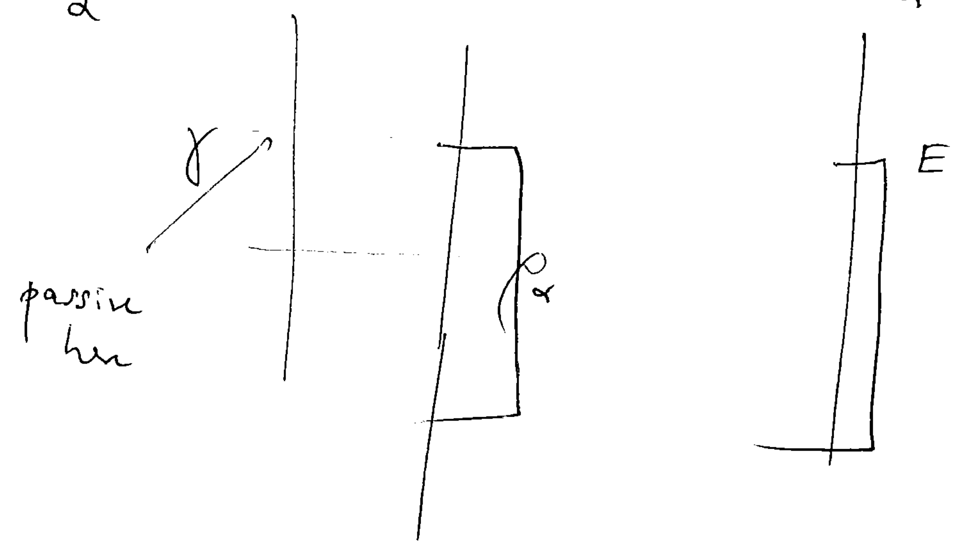
copy iteration of H to M, using original $\pi : H \longrightarrow M$.

<u>rules of comparison.</u>

compare B with itself.

the γ index least disagreement at stage α.

Say: $B_\alpha^{\mathcal{J}} = H_\alpha^{\mathcal{J}}$ $\quad J_\alpha^{\mathcal{J}}$ $\quad H_\alpha^{\mathcal{U}}$



$\Rightarrow$ $H_\alpha^{\mathcal{J}} \not\trianglelefteq R$, any $R = H_\gamma^{\mathcal{U}}$ or $R = J_\gamma^{\mathcal{U}}$ for $\gamma > \alpha$

then use E on $\mathcal{U}$-side, do nothing on $\mathcal{J}$-side but promise next ext. on $\mathcal{J}$-side is taken from $H_\alpha^{\mathcal{J}}$.

<u>Claim</u>. comparison terminates.

<u>Claim</u>. it doesn't.

- extenders applied to a single model are close to that model.
- preserve fine structure in standard manner.

So if $\alpha \notin \beta^{\mathcal{J}}$

$\beta <_{\mathcal{J}} \alpha$, no drop in model or degree in $(\beta, \alpha]_{\mathcal{J}}$

then $i_{\beta,\alpha}$ and $i^*_{\beta,\alpha}$ preserve fine struct in the usual manner

$i_{\beta,\alpha} : H_\beta \to H_\alpha$,

not sound

$\Rightarrow H_\alpha \not\trianglelefteq R$.