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recognizable sets of ordinals.

(joint with melvin calf)

 ordinal turing machines OTM

lim for x in state

1. p states \rightarrow states
2. content of the cell
3. head position : lin of

x \in x (OTM-) compute for x

(\rightarrow x) is an output of a computation which has x as an additional input.
\[
x \text{ complete } \iff x \in L.
\]

\[
x \text{ is recognizable if there is a program } P \text{ and some } \beta
\]

\[
\forall y \in x \\forall \gamma \in \beta \iff P(y, \beta) = 1 \iff x = y.
\]

Output of complete

\text{and input } y \text{ and } \beta \text{ as add. input}

without additional ordinal pair

\[
x \in w \text{ complete } \iff
\]

\[
x \text{ is } \beta \text{ recognizable } \iff
\]

\[
x \in L, \text{ where}
\]

\[
\gamma \text{ is least } \gamma \in L \leq \Sigma
\]

arises that \( M^\# \) exists.

\underline{Question}: is every recognizable \( \gamma \) of ordinals in \( L \)?
\( x \subset \alpha \) is in the recognizable closure if the \( x \nolimits \)
\( x = x_0, x_1, \ldots, x_n \) recognizable,
\( x \nolimits \) recognizable \( \Rightarrow x_{i+1} \).

**Lem.** Suppose that \( x \subset \alpha \). TFAE:

- \( x \) is constructible from \( y \subset \beta \)

1. \( y \) is recognizable.
2. There is an \( y \) such that \( y \subset \beta \)
   \( y \) is the unique subset \( z \) of \( \beta \) s.t.
   \( L(z) = \gamma(z, y) \).
3. \( y \) implicitly defines one \( L \)
   i.e. there is an \( y \) such that \( y \subset \beta \) s.t.
   \( y \) is the unique \( z \subset \beta \) s.t.
   \( (L, z, e, z) = \gamma(z, y) \).

\[ M^* = \text{the } \alpha^* \text{ chain of } M, \]
by the least witness cardinal
\( + \text{ its images} \)
\[ M^0 = \bigcap_{\alpha \in \omega} M^\alpha \]

lemma. if \( x \in x \) is recognizable, then \( x \in M^0 \), in \( \alpha < \omega \).

question. is every recognizable subset \( W \) in \( M_1 \)?

lemma. if \( P \) is homogenous, \( C \) is \( P \)-generic over \( V \), supp. that \( y \in C \) or is recognizable, \( y \in V \cup y \). then \( y \in V \).

lemma. supp. \( V \) ex. \( 2F + \omega \) has, eg. \( X \in y \), is in \( L(y) \), then \( y \in w \).

lemma. if \( P \) be homogenous, \( TP \) doesn't contain \( w \), \( TP \) from choice, then \( y \) recognizes sum \( y \) is in \( M_1 \).
question. supp. that $H_{\omega_2}$ is closed in $\mathcal{M}_1$?

then is any rec. subset of $\omega_1$ in $\mathcal{M}_1$?

queries: can we add recognizable sets by forcing using large cardinals?

less. (steel) supp. $\alpha < \omega_1$. 

say $\rho_\omega(N) = \omega$, $N$ has length $\alpha$, $N$ sound, $N$ is $\in \mathcal{M}_1$, then $N = H_{\omega_1 \alpha}$.

less. supp. that $\alpha < \omega_1^{\omega_1}$,

$\rho_\omega(M_{\omega_1 \alpha}) = \omega$.

in $y \subset w$ be a can. code for $M_{\omega_1 \alpha}$, then $y$ is recognizable.
then if \( x \in M_1 \parallel \alpha \),

\( x \) is constructible for \( y \).

\[ \Rightarrow M_1 \parallel w_1^M \quad \text{is recognized in} \quad w_1^M. \]