On spectral triples in quantum gravity

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Motivation:

- The formulation of the standard model in noncommutative geometry
- Canonical gravity, Loop Quantum Gravity

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Physical interpretation:

- The spectral triple encodes the information of the kinematical part of quantum gravity.
- The spectral triple has semi-classical states which gives the Dirac Hamiltonian in $3+1$ dimension.

The standard model and noncommutative geometry

(Connes, Lott, Chamsedine, Marcolli, ...)

$$(C^\infty(M) \otimes \mathcal{B}_F, L^2(M,S) \otimes \mathcal{H}_F, D \otimes 1 + \gamma_5 \otimes D_F)$$

where
M - 4-dimensional compact spin manifold
S - spin bundle
$\mathcal{B}_F$ - finite dimensional algebra
$\mathcal{H}_F$ - finite dimensional Hilbert space, fermionic content of the standard model
D_F - certain matrix

The standard model and noncommutative geometry

Spectral action:

$$I = \langle \psi | \tilde{D} | \psi \rangle + Tr\left(\varphi\left(\frac{\tilde{D}^2}{\Lambda^2}\right)\right)$$

action of standard model coupled to gravity.

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Main point

Formulation of the standard model as a single gravitational theory.

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action of standard model coupled to gravity.

Main point

Formulation of the standard model as a single gravitational theory.
Essentially classical, no quantization.

The standard model and noncommutative geometry

Question:

How to formulate a quantization procedure within noncommutative geometry?

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How to formulate a quantization procedure within noncommutative geometry?

Would involve quantum gravity.

Our aim:

To construct a model which combines noncommutative geometry with elements of quantum gravity.

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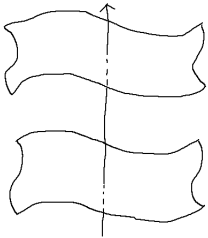
Loop Quantum Gravity:

Quantization of gravity. No unification.

Loop Quantum Gravity



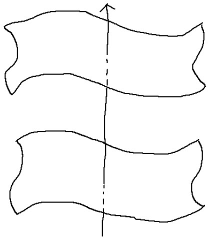
$$M = \mathbb{R} \times \Sigma$$

The new (Ashtekar) variables

A^i_j - $SU(2)$-connection on Σ.

$E^i_j = |\mathrm{det} e|^{\frac{1}{2}} e^i_j$ - e^i_j orthonormal frame field.

Loop Quantum Gravity



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Loop Quantum Gravity

Poisson bracket

$$\{A_j^i(x), E_l^k(y)\} = \delta_l^i \delta_j^k \delta(x - y)$$

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Constraints

Gauss constraint

$$\partial_i E^i_a + \epsilon^c_{ab} A^b_i E^i_c = 0$$

Diffeomorphism constraint

$$E^j_a F^a_{ij} = 0$$

Hamilton constraint (Euclidian)

$$\epsilon^{ab}_c E^i_a E^j_b F^c_{ij} = 0$$

Loop Quantum Gravity

Reformulation

L loop on Σ.

$$h_L(\nabla) = \mathrm{Hol}(L, \nabla)$$

∇ - $SU(2)$-connection on Σ.

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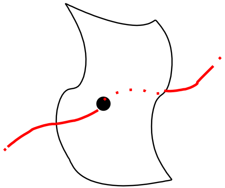
$$F_a^S(E) = \int_S \epsilon_{mnp} E_a^m dx^n dx^p$$

S - surface in Σ.

Loop Quantum Gravity



C Curve. Where $C = C_1 C_2$.

$$\{F_a^S(E), h_C(\nabla)\} = \pm h_{C_1}(\nabla)\tau_a h_{C_2}(\nabla)$$

τ^a generator of $\mathfrak{su}(2)$.

Loop Quantum Gravity

Quantization strategy

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$$[\hat{F}_a^S, \hat{h}_C] = \pm \hat{h}_{C_1} \tau^a \hat{h}_{C_2}$$

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Express constraints in terms of h_C, F_S^a and replace with $\hat{F}_a^S$ and $\hat{h}_C$.
Solve the quantum constraints to get the physical Hilbert space.

Our project part 2

G - connected compact Lie group.
$G \to M_N$ - matrix representation.
M - manifold.
x_0 - point in M.
$\mathcal{A}$ - space of connections in $M \times G$.
$h_L : \mathcal{A} \to M_N$ given

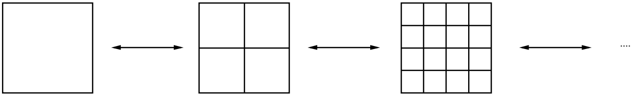
$$h_L(\nabla) = \mathrm{Hol}(L, \nabla), \quad \nabla \in \mathcal{A}.$$

Let $\mathcal{B}$ be the algebra generated by

$$\{h_L\}_{L \text{ based in } x_0}$$

We want to construct a spectral triple on $\mathcal{B}$.

Completing spaces of connection



Γ_0 lattice on M.
Γ_n the n'th subdivision of Γ_0.
Identify

$$\mathcal{A}_{\Gamma_n} = G^{e(\Gamma_n)}$$

via

$$\mathcal{A}_{\Gamma_n} \ni \nabla \to (Hol(e_1, \nabla), \ldots, Hol(e_{e(\Gamma_n)}, \nabla)),$$

where $e(\Gamma_n)$ is the number of edges in Γ_n.

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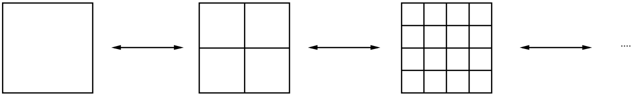

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where $e(\Gamma_n)$ is the number of edges in Γ_n.
When n tends to ∞, $\mathcal{A}_{\Gamma_n}$ will be a good approximation to $\mathcal{A}$.

Completing spaces of connection

There are maps

$$P_{n+1,n} : \mathcal{A}_{n+1} \to \mathcal{A}_n.$$

Define

$$\overline{\mathcal{A}}^s = \lim_n \mathcal{A}_n$$

Topology on $\mathcal{A}_n = G^{e(\Gamma_n)}$ induces topology on $\overline{\mathcal{A}}^s$.

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Topology on $\mathcal{A}_n = G^{e(\Gamma_n)}$ induces topology on $\overline{\mathcal{A}}^s$.
It is not hard to see

$$\mathcal{A} \hookrightarrow \overline{\mathcal{A}}^s \text{ densely}$$

Define

$$L^2(\overline{\mathcal{A}}^s) = \lim_n L^2(\mathcal{A}_n)$$

The Ashtekar-Lewandowski case

M real analytic.

$$S_a = \{ \text{ finite graphs with piecewise analytic edges } \}$$

S_a is a directed set.
Define

$$\mathcal{A}_\Gamma = G^{e(\Gamma)},$$

where $\Gamma \in S_a$, and define

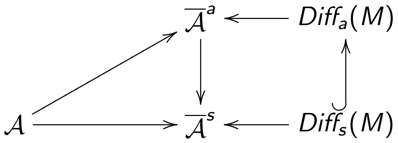
$$\overline{\mathcal{A}}^a = \lim_{\Gamma \in S_a} \mathcal{A}_\Gamma$$

$$L^2(\overline{\mathcal{A}}^a) = \lim_{\Gamma \in S_a} L^2(\mathcal{A}_\Gamma)$$

$L^2(\overline{\mathcal{A}}^a)$ is not separable.

Comparision

We have the following diagram



where $Diff_a(M)$ is the group of piecewise analytic diffeomorphisms, and $Diff_s(M)$ is the group of diffeomorphisms preserving the infinite lattice.

Comparision

We have the following diagram

$$\begin{array}{ccccc} & & \overline{\mathcal{A}}^{a} & \longleftarrow & \mathit{Diff}_a(M) \\ & \nearrow & \downarrow & & \uparrow \\ \mathcal{A} & \longrightarrow & \overline{\mathcal{A}}^{s} & \longleftarrow & \mathit{Diff}_s(M) \end{array}$$

where $\mathit{Diff}_a(M)$ is the group of piecewise analytic diffeomorphisms, and $\mathit{Diff}_s(M)$ is the group of diffeomorphisms preserving the infinite lattice. We would therefore like to see $\overline{\mathcal{A}}^{s}$ as $\overline{\mathcal{A}}^{a}$ subjected to a partial gauge fixing of the diffeomorphism group.

The algebra

$\mathcal{B}^s$ is the algebra generated by $\{h_L\}$, where L is a loop in $\cup\Gamma_n$ based in x_0.
$\mathcal{B}^s$ admits a representation on $L^2(\overline{\mathcal{A}}^s) \otimes M_N$.

The Dirac operator

Idea

$\mathcal{A}_\Gamma = G^n$ is a classical geometry and therefore has a Dirac operator. We take one acting on

$$L^2(G^n, Cl(T^* G^n)) \otimes M_N.$$

The maps $P_{i+1,i} : G^{n_{i+1}} \to G^{n_i}$ induces

$$P^*_{i+1,i} : L^2(G^{n_i}, CL(T^* G^{n_i})) \to L^2(G^{n_{i+1}}, CL(T^* G^{n_{i+1}}))$$

To ensure that $\{D_i\}$ descends to an operator D on

$$\lim_i L^2(G^{n_i}, CL(T^* G^{n_i})) \otimes M_N = L^2(\overline{\mathcal{A}}^s, CL(T^* \overline{\mathcal{A}}^s)) \otimes M_N,$$

we need to ensure

$$P^*_{i+1,i} \circ D_i = D_{i+1}$$

The Dirac operator

Restrict for simplicity to the case of a single edge.
Gives the projective system

$$G \leftarrow G^2 \leftarrow G^4 \leftarrow \ldots \leftarrow G^{2^n} \leftarrow \cdots$$

with structure maps

$$P_{n+1,n}(g_1, \ldots, g_{2^{n+1}}) = (g_1 g_2, \ldots, g_{2^{n+1}-1} g_{2^{n+1}}).$$

Can be rewritten to a projective system with structure maps

$$P_{n+1,n}(g_1, \ldots, g_{2^{n+1}}) = (g_1, \ldots, g_{2^n}).$$

The Dirac operator

Define

$$D_n = \sum_i a_i D_{0i}$$

where D_{0i} is a Dirac operator on the i'the copy of G.
Take D_{0i} of the form

$$D_{0i} = \sum_k e_k \cdot d_{e_k},$$

where $\{e_k\}$ denotes a orthonormal basis in $\mathfrak{g}$ and the corresponding left translated vectorfields.

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where $\{e_k\}$ denotes a orthonormal basis in $\mathfrak{g}$ and the corresponding left translated vectorfields.
The family $\{D_n\}$ is a consistent family of operator and hence descends to an operator D.

Semifiniteness

D does not have compact resolvent.

Definition

Let $\mathcal{N}$ be a semifinite von Neumann algebra with a semifinite trace τ. Let $\mathbb{K}_\tau$ be the τ- compact operators. A semifinite spectral triple $(\mathcal{B}, \mathcal{H}, D)$ is a $*$-subalgebra $\mathcal{B}$ of $\mathcal{N}$, a representation of $\mathcal{N}$ on the Hilbert space $\mathcal{N}$ and an unbounded densely defined self adjoint operator D on $\mathcal{H}$ affiliated with $\mathcal{N}$ satisfying

1. $b(\lambda - D)^{-1} \in \mathbb{K}_\tau$ for all $b \in \mathcal{B}$ and $\lambda \notin \mathbb{R}$..
2. $[b, D]$ is densely defined and extends to a bounded operator.

Semifiniteness

Rewrite

$$L^2(\overline{\mathcal{A}}^s, CL(T^*\overline{\mathcal{A}}^s)) \otimes M_N = (L^2(\overline{\mathcal{A}}^s) \otimes M_N) \otimes Cl(T^*_{id}\overline{\mathcal{A}}^s).$$

Let $\mathcal{N}$ be the weak closure of

$$\mathbb{B}(L^2(\overline{\mathcal{A}}^s) \otimes M_N) \otimes C,$$

where

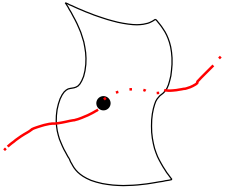
$$C = Cl(T^*_{id}\overline{\mathcal{A}}^s) = \lim_n Cl(T^*_{id}\mathcal{A}_n).$$

$\mathcal{N}$ is semi finite.

Theorem

When $a_i \to \infty$ the triple $(\mathcal{B}^s, D, L^2(\overline{\mathcal{A}}^s, Cl(T^*\overline{\mathcal{A}}^s) \otimes M_n))$ is semi finite with respect to $\mathcal{N}$

Poisson structure



C Curve. Where $C = C_1 C_2$.

$$\{F_a^S(E), h_C(\nabla)\} = \pm h_{C_1} \tau_a h_{C_2}$$

τ_a generator of $\mathfrak{su}(2)$. In the quantization setting C_1 and C_2 corresponds copies of G. Hence we look at $L^2(G^2) \otimes M_N$ and

$$\hat{h}_{C_1 C_2}(\xi)(g_1, g_2) = g_1 g_2 \xi(g_1, g_2).$$

With

$$\hat{F}^S = \mathcal{L}_{L_{g_1} \tau_a}$$

we have

$$[\hat{F}_a^S, \hat{h}_{C_1 C_2}] = \hat{h}_{C_1} \tau_a \hat{h}_{C_2}.$$

Therefore D in a certain sense contains quantization.

Semi-classical states

Let $\psi(x)$ be a spinor field on Σ and let $A(x)$ and $E(x)$ be a $SU(2)$-connection and a triad field on Σ.
We will now construct states that are localized around ψ, A, E to get a physical interpretation of D, our Dirac type operator.

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- **On one edge** ϵ**.** $\phi^t \in L^2(SU(2), M_2)$ coherent state **Hall 1994** with

$$
\begin{aligned}
\lim_{t\to 0}\langle\phi^t|f_\epsilon|\phi^t\rangle &= Hol(\epsilon, A) \\
\lim_{t\to 0}\langle\phi^t|td_{e_\epsilon^a}|\phi^t\rangle &= i2^{-2n}E_a^1(v_2),
\end{aligned}
$$

where v_2 is the endpoint of ϵ and the 1 in E_a^1 is the direction of ϵ.

Consider the state

$$\Psi(g) = (g\psi(v_2) + ie^a_\epsilon \sigma^a \psi(v_1))\phi^t_\epsilon(g).$$

A computation gives

$$\begin{aligned}\lim_{t\to 0}\langle\Psi|tD_\epsilon|\Psi\rangle &= a_n 2^{-2n}(-\bar{\psi}(v_1)\sigma^a E^1_a(\psi(v_1)-\psi(v_2)) \\ &\quad +(\bar{\psi}(v_2)-\bar{\psi}(v_1))\sigma^a E^1_a \psi(v_1) \\ &\quad +\bar{\psi}(v_1)\{2^{-n}A_1, \sigma^a E^1_a\}\psi(v_1))\end{aligned}$$

where we have used $g \sim 1 + 2^{-n}A_1$.

Consider the state

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$$a_n 2^{-2n}(\psi(v_1) - \psi(v_2)) \to \partial_1 \psi(v_1)$$

when $n \to \infty$, hence $a_n = 2^{3n}$.

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when $n \to \infty$, hence $a_n = 2^{3n}$. Then

$$\lim_{n\to\infty}\lim_{t\to 0}\langle\Psi|tD_\epsilon|\Psi\rangle = \bar{\psi}(v_1)(\sigma^a E^1_a \nabla_1 + \nabla_1 \sigma^a E^1_a)\psi(v_1),$$

where we have used partial integration and $\nabla = d + A$. This is the expression for the Dirac operator in 3 dimension in the 1 direction.

Change of basepoint

It turns out that to do this for all edges is related to the choice of basepoint.
L_0 - loop based in x_0.
p - path from x_0 to x_1, $p = \{l_1, \ldots, l_n\}$.
U_p - parallel transport along p.

$$h_{L_{x_1}} = U_p h_{L_{x_0}} U_p^*$$

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Change of basepoint

Lift U_p to

$$\tilde{U}_p = \tilde{U}_1 \cdots \tilde{U}_n,$$

where

$$\tilde{U}_i = ie_i^a (g_i \otimes \beta_i^a),$$

where β_i^a are skew self-adjoint matrices satisfying

$$\sum_a |\beta_i^a|^2 = 1.$$

For $p_1 \neq p_2$,

$$\langle \tilde{U}_{p_1} | \tilde{U}_{p_2} \rangle = 0$$

(Here we have to tensor the Hilbert space with an extra matrix factor.)

Change of basepoint

Let $\psi(v_i)$, $v_i \in \Gamma_n$ transform

$$\tilde{U}_{p_i}\psi(v_i),$$

where p_i is a path from x_0 to v_i. We have

$$\begin{aligned}&\langle \tilde{U}_{p_1}\psi(v_1) + \tilde{U}_{p_2}\psi(v_2)|h_{L_0}|\tilde{U}_{p_1}\psi(v_1) + \tilde{U}_{p_2}\psi(v_2)\rangle \\ &= \langle \psi(v_1)|h_{L_1}|\psi(v_1)\rangle + \langle \psi(v_2)|h_{L_2}|\psi(v_2)\rangle.\end{aligned}$$

L_1 loop based in v_1. L_2 loop based in v_2.

Change of basepoint

To eliminate this choice of base points we sum over all of them

$$\Psi_n = \frac{1}{N} \sum_i \tilde{U}_{p_i} \psi(v_i)$$

and define

$$\Psi_n^t = \Psi_n \prod_{e \in \Gamma_n} \phi_e^t$$

The expectation value of the Dirac operator

Set

$$\beta_i^a = N(v_i)\gamma^a + iN^a(v_i)\gamma^0,$$

where N, N^a are lapse and shift fields. A computation gives

$$\begin{gathered}\lim_{n\to\infty} \lim_{t\to 0} \langle \Psi_n^t | tD | \Psi_n^t \rangle \\ = \int_\Sigma \bar{\psi}(x)(\sqrt{g} e_a^m \nabla_m + \nabla_m \sqrt{g} e_a^m)(N(x)\gamma^a + iN^a(x)\gamma^0)\psi(x) dx \\ + \text{lower order terms.}\end{gathered}$$

This expression resembles the Dirac hamiltonian in $3+1$-dimension.
Thus the semi-classical states can be interpreted as one fermion states in a background gravitational field with lapse N and shift N^m.
Hence D can be interpretated as a quantization of the Dirac Hamiltonian.

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- Construct many particle states.

Outlook/Problems

- Compute quantum fluctuation of the semi-classical limit.
- Construct many particle states.
- Need more structure than just a spectral triple to make contact with the standard model. Real structure,...In work in progress it look like getting the matrix factor of the γ-matrices right automatically gives rise to part of the structure the standard model (the real structure.) The expectation value of a Loop operator looks like a matrix valued function on Σ.

Thank you for your attention