Synthesis of Petri Nets with Whole-place Operations and Localities

Jetty Kleijn  Maciej Koutny  Marta Pietkiewicz-Koutny
Leiden University  Newcastle University  Newcastle University
The Netherlands  United Kingdom  United Kingdom

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Outline

- motivation

- synthesising Petri nets from step transition systems
  - synthesis problem
  - net-types and regions
  - net-type for Petri Nets with Whole-place Operations and Localities (WPOL-nets)
  - solving synthesis problems (feasibility and effective construction)

- future work
• automated synthesis from behavioural specifications: approach for constructing computational systems
• guarantees that the resulting systems are correct by design
• HERE: systems are represented by Petri nets
• HERE: specifications are labelled transition systems
• growth area: “mining” processes from logs of observed behaviour
Example: Petri net
Executing a Petri net

by firing **steps** (multisets of transitions) we can move from one marking to another generating the **concurrent reachability graph (CRG)** of a net.
modified example: one producer and two consumers
consumers are co-located
locally maximal execution semantics (step firing policy)
step \{c1\} is illegal
\{c1, c4\} and \{c1, c4, p1\} are legal
• **Ordinary Petri Nets:** unannotated arcs carry one token per firing
• **PT-nets:** arcs annotated with \( n \geq 2 \) carry \( n \) tokens per firing
• **Petri Nets with Whole-place Operations:** arcs annotated with linear expression involving places carry (per firing) variable numbers of tokens determined by the current marking
Expressiveness

Observation scenario with parameter $X$

No standard Petri net can model this scenario
Expressiveness

- It’s a solution
- Arc weights depend on current markings
- Linear annotations
nets with localities can be used to describe and analyse globally asynchronous locally (maximally) synchronous systems (GALS)

examples:
- VLSI chips with multiple clocks for synchronisation of different subsets of gates
- membrane systems modelling cells inside which reactions are carried out in co-ordinated pulses
nets with whole-place operations can model:

- message passing interface (MPI) programs that perform parallel computations in the environment of distributed memory

- biological systems where the activation of reactions depends on the relative concentration of specific molecules and catalysts
Step transition systems

- behavioural model for WPOL-nets

- step transition system $\text{TS} = \langle Q, \mathbb{N}^T, \delta, q_0 \rangle$:
  - $Q$ states
  - $q_0 \in Q$ initial state
  - $\delta: Q \times \mathbb{N}^T \rightarrow Q$ partial function describing arcs labelled by multi-sets of (net) transitions from $T$ executed concurrently

- assumption: every state $q \in Q$ is reachable from $q_0$

- $\text{TS}$ is bounded if there is finite number of outgoing arcs at its every state
- $\text{TS}$ is finite if it is bounded and has finitely many states
General synthesis problem

TS  given step transition system
\( \mathcal{N} \)  Petri net obtained from TS through synthesis procedure

**Aim:** isomorphism between \( \text{CRG}(\mathcal{N}) \) and TS
Regions and net-types

- one needs to construct enough net places using the information contained in $TS$
- suitable places are represented by regions, which depend on parameter $\tau$ (net-type)
- $\tau = \langle Q, S, \Delta \rangle$ is an uninitialized transition system over an abelian monoid $S$ capturing the behaviour of a place of a net of a particular type
  - $Q$ - values that can be stored in a place
  - $S$ - the nature of connections between a place and net transitions
  - $\Delta$ - gives the enabling conditions and the newly generated values for steps of transitions
- net-type for PT-nets: $\tau_{PT} = \langle \mathbb{N}, S_{PT}, \Delta_{PT} \rangle$, where $S_{PT} = \mathbb{N} \times \mathbb{N}$ and $\Delta_{PT}(n,(i,o)) = n - i + o$ provided that $n \geq i$
k-restricted WPOL-nets (k-WPOL-nets)

- **problem**: change of markings in WPOL-nets depends on current markings
- instead of constructing individual places we need to construct **clusters of related** places
- two places are **related** if (at least) one of them is a **whole-place** used in the annotations of arcs adjacent to the other place
- we partition places of a net into clusters (of no more than k places), with no exchange of whole-place marking information between different clusters
- **k-WPOL-nets**: WPOL-nets with a set of places partitioned into clusters of no more than k **related places**
- every k-WPOL-net can be expressed as a WPOL-net
• a class of nets can be expressed as a class of $\tau$-nets if we find a suitable net-type $\tau$ to describe its places
• k-WPOL-nets are not $\tau$-nets according to the original definition
• extended net-type capturing the behaviour of sets of $k$ places:

$$\tau^k = \langle \mathbb{N}^k, (\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k, \Delta^k \rangle,$$

where

$$\Delta^k : \mathbb{N}^k \times ((\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k) \rightarrow \mathbb{N}^k$$

matrices specifying the nature of connection between a cluster of places and a transition

generates vectors specifying the number of tokens in a cluster of $k$ places
Problem 1 (feasibility)

TS is a bounded step transition system, $k > 0$, and $\ell$ is a locality mapping for transitions in TS

Provide necessary and sufficient conditions for TS to be realised by some $\tau^k$-net $\mathcal{N}$ executed under the locally maximal step firing policy defined by $\ell$, i.e. $TS \cong \text{CRG}_\ell(\mathcal{N})$

Problem 2 (effective construction)

TS is a finite step transition system, $k > 0$, and $\ell$ is a locality mapping for transitions in TS

Decide whether there is a finite $\tau^k$-net, realizing TS when executed under the locally maximal step firing policy defined by $\ell$. If the answer is positive construct such a $\tau^k$-net
solving Problem 1: define a $\tau^k$-region of $TS = \langle Q, \mathbb{N}^T, \delta, q_0 \rangle$ as two mappings $\langle \sigma, \eta \rangle$:

\[
\sigma : Q \rightarrow \mathbb{N}^k \\
\eta : T \rightarrow (\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k
\]

such that, for every state $q \in Q$ and every step $\alpha$ enabled at $q$ in $TS$,

\[
\eta(\alpha) = \sum_{t \in T} \alpha(t) \cdot \eta(t)
\]

is enabled at $\sigma(q)$ in $\tau^k$ and $\Delta^k(\sigma(q), \eta(\alpha)) = \sigma(\delta(q,\alpha))$
Separation axioms

- regions of TS allow no less behaviour than TS, but they can allow more!
- to construct a net realising TS, we must find enough regions, so that the following hold:
  - Axiom 1 (state separation): for any states $q \neq r$ of TS, there is a $\tau^k$-region $<\sigma, \eta>$ of TS that distinguishes these states ($\sigma(q) \neq \sigma(r)$)
  - Axiom 2 (forward closure): for every step $\alpha$ that is not enabled in TS at some state $q$ one of the following holds:
    - $\alpha$ is not region enabled: there is a $\tau^k$-region $<\sigma, \eta>$ of TS such that $\eta(\alpha)$ is not enabled at $\sigma(q)$ in $\tau^k$
    - $\alpha$ is not control enabled: $\alpha$ is rejected by the locally maximal step firing policy
TS can be realised by a $\tau^k$-net under the locally maximal step firing policy associated with $\ell$

iff

Axioms 1 & 2 are satisfied
• to solve Problem 2 using Main Result one needs to find an effective representation of $\tau^k$-regions of TS
• we define system $S_{TS}$ of equations and inequalities encoding the conditions to be satisfied by $\tau^k$-regions
• the non-negative solutions of $S_{TS}$ are in one-to-one correspondence with $\tau^k$-regions of TS
• one can then check Axioms 1 & 2 for them

• for PT-nets, a similar procedure leads to a homogeneous linear system such that one can always find a (sufficiently rich) finite basis for all the solutions (regions)

• for $k$-WPOL-nets, $S_{TS}$ is (unfortunately) quadratic
we consider sub-problem, Problem 3, assuming that we know all the whole-places of the net to be synthesised with their markings at every state of TS (no information about their connections to transitions yet!)

as TS is finite, we have finite number of locality mappings to explore one-by-one, and so we can assume that $\ell$ is fixed

knowing the markings of whole-places, we can deduce the ranges of the coefficients in the annotations of the arcs connecting these places with transitions from $T$, hence we can investigate them set-by-set, and so we can assume they are fixed

every instance of a net $\mathcal{N}$ obtained as above (with only whole-places) can be checked whether it realizes TS
Adding non-whole-places

- If $\mathcal{N}$ does not realise TS we add non-whole-places to $\mathcal{N}$
- We build a modified system $S'_TS$ to discover $k$-tuples of places with $m$ whole-places and 1 generic non-whole-place ($k = m + 1$)
- $S'_TS$ with many variables turned into concrete values due to the known information about whole-places is linear

\[
[ m^q(p_1), \ldots, m^q(p_m), m^q(p), 1 ] \ast [ \text{coeff}_1, \ldots, \text{coeff}_m, 0, \text{coeff}_0 ]
\]

vector specifying the coefficients in the annotation of the arc between non-whole-place $p$ and some transition $t$
• each solution of $S'_{TS}$ yields one non-whole-place

• one can always find a finite basis for all the solutions of $S'_{TS}$

• we can add the basis solutions (regions) to the set of regions corresponding to the whole-places and check Axioms 1 & 2

• if they are satisfied, the net obtained by adding extra non-whole-places to $\mathcal{N}$ is a solution to Problem 3; otherwise, there is no solution
Future work

- use the developed theory in selected case studies

- investigate the relationship between the locality mapping and the grouping of the places into clusters in k-WPOL-nets

- develop a synthesis approach for WPO-nets executed under more general step firing policies:
  - based on linear rewards of steps, where the reward for firing a single transition is fixed
  - based on linear rewards of steps, where the reward for firing a single transition depends on the current net marking
Thank you!