Bibliographic references for
KK-theory and its applications

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1 Preliminaries

1.1 C*-algebras

You can consult the books by Murphy [Mur90] or by Wegge-Olsen [WO93] for the following definitions and results which will be used throughout the lecture:

C*-algebras, Gelfand theorem, *-homomorphisms, ideals and quotients of C*-algebras, injective *-homomorphisms are isometric, *-homomorphisms have closed image, approximate units, σ-unitality, (separable) C*-algebras admit (countable) approximate units, multiplier algebras.

1.2 K-theory for C*-algebras

We will meet K-theory at several places during the lectures and see that $\mathrm{KK}(\mathbb{C}, B) \cong \mathrm{K}_0(B)$ for all C*-algebras B. If you don't really care you can take this as a definition, but to understand what is going on it might be worthwile to understand what K-theory is. The book by Wegge-Olsen [WO93] gives a detailed introduction to K-theory for C*-algebras but you can also look into the book by Blackadar [Bla98] to make yourself acquainted with the following topics:

The definitions of K_0 (using projections) and K_1, the six-term exact sequence, Bott periodicity, the definition of K_0 using modules, the relation to vector bundles (i.e. $\mathrm{K}_0(\mathcal{C}_0(X)) \cong \mathrm{K}^0(X)$ for any compact Hausdorff space X).

1.3 Fredholm operators

For Fredholm operators between Hilbert spaces you can consult your favourite book on functional analysis (or even Wikipedia). You just need to know (for a start) what the definition of a Fredholm operator and its index is. To know some elementary properties of the index (additivity, homotopy invariance,...) is helpful.

1.4 Clifford algebras

I will give the definition of complex Clifford algebras in the lecture, but some background knowledge might be useful. Blackadar's book [Bla98] does certainly contain everything that we need. To get a feeling for Clifford algebras you can also read a bit on Clifford algebras of quadratic forms over arbitrary fields; any advanced general algebra book will contain a section on Clifford algebras (the book by Lang being an example). If you are really keen, you can search the respective section in Bourbaki. As an introduction, however, just read the article on Wikipedia...

2 Kasparov's picture of KK-theory

2.1 Hilbert modules and adjoinable/compact operators

We will discuss Hilbert modules and operators between them at the beginning of the lecture series. What we do is contained in [WO93] or in [Bla98], but you can also consult [JT91].

2.2 Kasparov's stabilization theorem

I guess that this is explained in [Bla98] as well as in [JT91].

2.3 Graded C*-algebras and graded modules

We will spend some time on this at the beginning and follow [Bla98].

2.4 Kasparov cycles

The definitions and basic properties of Kasparov cycles are given in [Bla98] and [JT91]. We will of course discuss different types of homotopies, the pullback and the pushforward of cycles, the sum of cycles etc..

3 The Kasparov product

Here, we will mainly follow the exposition in [JT91], but you can also have a look at [Bla98]. In addition, you can find a detailed exposition of the topic in the publications section of my web page.

References

[Bla98] Bruce Blackadar. *K-theory for operator algebras*, volume 5 of *Mathematical Sciences Research Institute Publications*. Cambridge University Press, Cambridge, second edition, 1998.

[JT91] Kjeld Knudsen Jensen and Klaus Thomsen. *Elements of KK-Theory*. Birkhäuser, 1991.

[Mur90] Gerard J. Murphy. *C^*-algebras and operator theory*. Academic Press Inc., Boston, MA, 1990.

[WO93] Niels E. Wegge-Olsen. *K-theory and C^*-algebras: a friendly approach*. Oxford University Press, 1993.