

Math 3U3, Test 1
Bradd Hart, Feb. 7, 2013

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (5 marks) An integer n is square-free if n is not divisible by m^2 for any $m > 1$. How many square-free integers n are there such that $1 \leq n \leq 100$?
2. (5 marks) Show that $r(3,4) \leq 10$.
3. (5 marks) Prove that for every $n > 1$,

$$\binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} + \cdots + (-1)^{n-1}n\binom{n}{n} = 0$$

4. We say that a set of circles on the surface of a sphere is in general position if every two distinct circles from the set intersect in exactly two points and no three distinct circles have a common point.
 - (a) (5 marks) Develop a formula for the number of regions on the surface of a sphere determined by n circles in general position.
 - (b) (Bonus, 2 marks) Define what it means that n spheres are in general position and develop a formula for the number of regions of 3-space determined by n spheres in general position.