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Universität Münster
Wintersemester 2022/23

Geometric Group Theory I
Exercise Sheet 11

Let G be a free group. An element $g \in G$ is called a *primitive* element in G, if g is contained in a set $X \subseteq G$ such that G is free with basis X.

Exercise 1. Let G be a free group and g a primitive element in G. Let $H \leq G$ be a subgroup containing g. Show that g is a primitive element in H.
Hint: Use the proof of the Kurosh Theorem (Theorem 7.52 in the lecture notes).

(4 Points)

Exercise 2. Show the following:

a) A finitely generated torsion group is FA.
b) If G is FA, then any quotient of G is also FA.
c) Let H be a normal subgroup of G. Suppose H and G/H are FA, then G is FA.
d) Let H be a subgroup of G of finite index. Suppose H is FA, then G is FA.[1]

(8 Points)

Exercise 3. Let σ be an automorphism of a tree X with no fixed points. Let $m := \inf\{d(x, \sigma(x)) : x \in X^0\}$ where $d(x, \sigma(x))$ is the length of the geodesic from x to $\sigma(x)$ in X. Let Γ be the group generated by σ. Show that Γ acts freely on X and the quotient graph ${}_{\Gamma}\backslash^{X}$ contains exactly one circuit, and this circuit is of length m.

(4 Points)

Exercise 4. *(Bonus exercise, 4 bonus points)* Let G be a group acts non-inversively on a connected non-empty graph X. Let $(\mathcal{G}, Y)$ be the quotient graph of groups with T a maximal subtree of Y. Let $\widetilde{Y}$ be the universal cover of $(\mathcal{G}, Y)$. Show that the map $\psi : \widetilde{Y} \to X$ (defined on page 43 of the lecture notes) is locally surjective. Conclude that $\widetilde{Y}$ is the universal cover of X.
Hint: First show that it is enough to show that ψ is surjective from the star of $\widetilde{x}$ in $\widetilde{Y}$ to the star of $\widehat{x}$ in X for all $x \in T^0$.

*Submission by **Wednesday** morning 11:00, **25.01.2023**, in Briefkasten 161.*
The exercise sheets should be solved and submitted in pairs.
Tutorial: Fridays 12:00-14:00, in room SR1d.
If you have questions about the problem sheet, please write to Tingxiang: tingxiangzou@gmail.com.

[1] Remark: The converse of this statement is not true.