farmer schlutzenberg, july 17

<u>scales</u>

φ fmla. $A_\varphi = \{ x \in \mathbb{R} : \exists x\text{-mouse } M \models \varphi \}$

e.g. $\varphi =$ "i am $x^\#$ and ψ."

$\uparrow$ e.g. Σ_1

write M_x least M s.t. $x \in A_\varphi$.

<u>truth norms.</u>

for fmla φ, 0–1 norm Φ_φ :

$$\Phi_\varphi(x) = \begin{cases} 0, & \text{if } M_x \models \varphi \\ 1, & \text{o.w.} \end{cases}$$

if $\{x_n\} \subset A_{\varphi_0}$ and $x_n \to x$ mod Φ

$M_\infty =$ unique pt.wise defl. $M \models T_\infty = \{ \varphi : \forall^* n \; M_{x_n} \models \varphi \}$

$=$ un($(M_{x_n})_{new}$, Φ_{conv})

using functions $f_t : \omega \to \bigcup M_{x_n}$

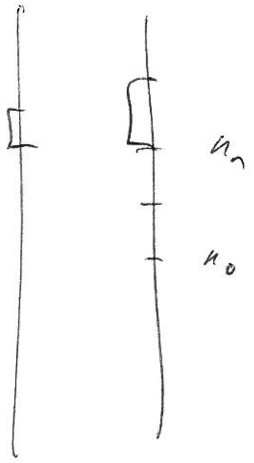
$n \mapsto t^{M_{x_n}}$, t a term, no variables

<u>lifting norms.</u>

for each <u>ordinal term</u> t (outputs $\in$ OR)

$= t(v_0, \ldots, v_{n-1})$

defn a norm $x \leq_t y$:



$(z^\#)_n = n^{th}$ il. of $z^\#$, $z \geq_T x, y$

κ_i indiscernible

N_x = output of background constr. on x in $(z^\#)_n$,

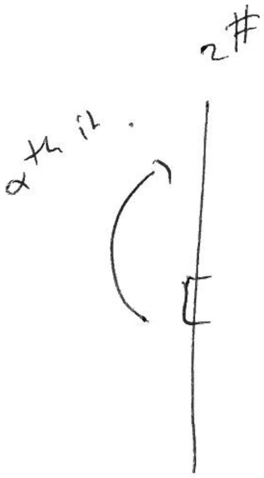
Same for x

defn norm $x \leq_t y$ iff for some/all $z \geq_T x, y$ $\quad t^{N_x}(\kappa_0 - \kappa_{n-1}) \leq t^{N_y}(\kappa_0 - \kappa_{n-1})$

say $x_n \to x$ mod all norms

<u>claim</u>. $M_\infty = x^\#$; say $N = \alpha^{th}$ il. of M_∞.

pick $z \geq_T ((x_n), x)^\#$

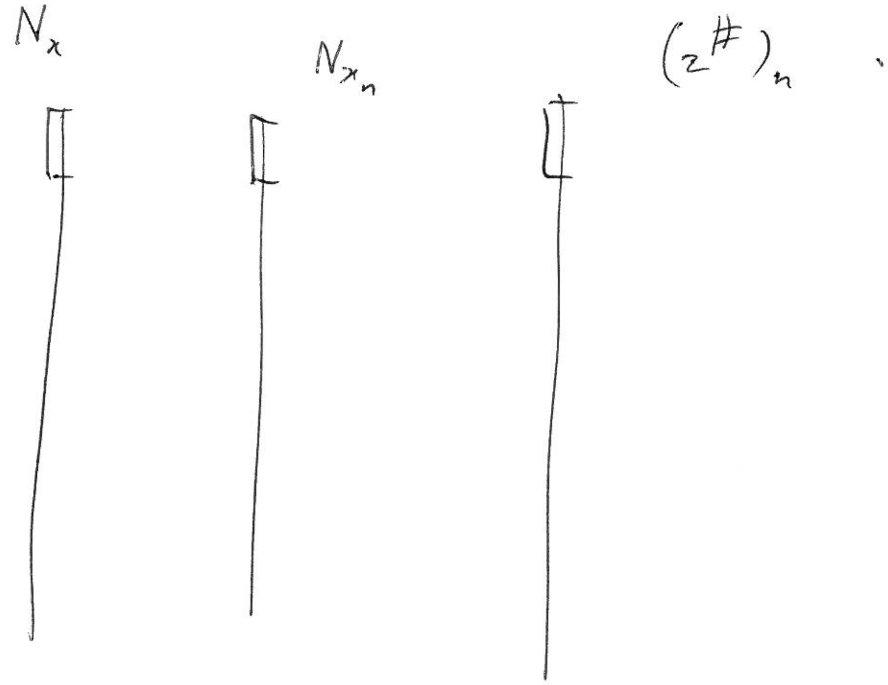


$P = \alpha^{th}$ il. of $z^\#$

map $t^N(\kappa_{\alpha_1}, \dots, \kappa_{\alpha_k})$ to

$$\lim_{n\to\infty} t^{N^P_{x_n}}(\iota_{\alpha_1}, \dots, \iota_{\alpha_k}) .$$

lower semi-continuity



work in $(z^\#)_n$.

let $C = \{\alpha < \kappa : \alpha$ is a crit. pt. of the $x_n^\#$-iteration for all $n\}$

note C is closed and $C \cap \kappa_i$ is unbdd. for each i.

let $C = \{c_\alpha\}_{\alpha<\kappa}$.

let $\{\mu_\alpha\}_{\alpha<\kappa}$ enum x-indiscernibles $< \kappa$.

$$\pi : \underset{\substack{\shortparallel \\ (x^\#)_n}}{N_x \cap OR} \longrightarrow OR,$$

$$\pi(t^{N_x}(\mu_\alpha, \dots)) = \lim_{n\to\infty} t^{N_{x_n}}(c_\alpha, \dots).$$

π order preserving

$\pi(\kappa_i) = \kappa_i$

$\pi(\alpha) \geq \alpha$:

$$\begin{array}{c} \pi(\alpha) \\ \parallel \\ \lim_{n\to\infty} t^{N_{x_n}}(\kappa_1,\kappa_2,\kappa_3) \geq t^{N_x}(\kappa_1,\kappa_2,\kappa_3) \end{array}$$

$\Rightarrow$ lower semi continuity.

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plan. ① $\mathbb{C}$ = construction

② define norms

③ prove that they are indeed norms

④ iterability of M_∞

⑤ lower semi continuity.

① $\mathbb{C}$ in a mouse Y (= a mouse over a real)

backgrounding : $(N_\alpha : \alpha \leq \Omega)$

$t_\alpha = \emptyset$.

type I. given N_α, if there's s.t.

$(N_\alpha; E)$ is an active pm,

and $\exists E^* \in \mathbb{E}^Y$ = ext. seq. of Y,

and $\nu(E^*) = \dot{N}_\alpha^Y$, then $N_{\alpha+\omega} = (N_\alpha; E)$

with E^* having min. index.

$t_\delta = 3$

<u>type III</u>. Q – local backgrounding

given Υ – card. δ s.t.

$J(\Upsilon | \delta) \models \delta$ woodin.

then $Q = \begin{cases} Q\text{ – str. for } \delta, & \text{or} \\ \Upsilon \end{cases}$

given N_δ, $(N_\alpha)_{\alpha \in [\delta+\omega, OR^Q]}$ is P – constr.

of Υ over N_δ, using all ext's for Q.

(then: this works, $OR^{N_\alpha} = \alpha$,

N_α is sound for $\alpha < OR^Q$,

N_{OR^Q} is Q – structure for δ.

(or $Q = \Upsilon$).

$N_\beta \models$ woodin exists

($\Rightarrow$ β is in same P – constr. interval.)

$t_\kappa = 2$

<u>type 2</u>. given κ measurable in Υ, given

N_κ (then $N_\kappa \cap OR = \kappa$),

then add the Jensen stack

(some pm s.t. $N_h \lhd P$,
$\rho_\omega^P = 1$, $P \models$ "cardinal")

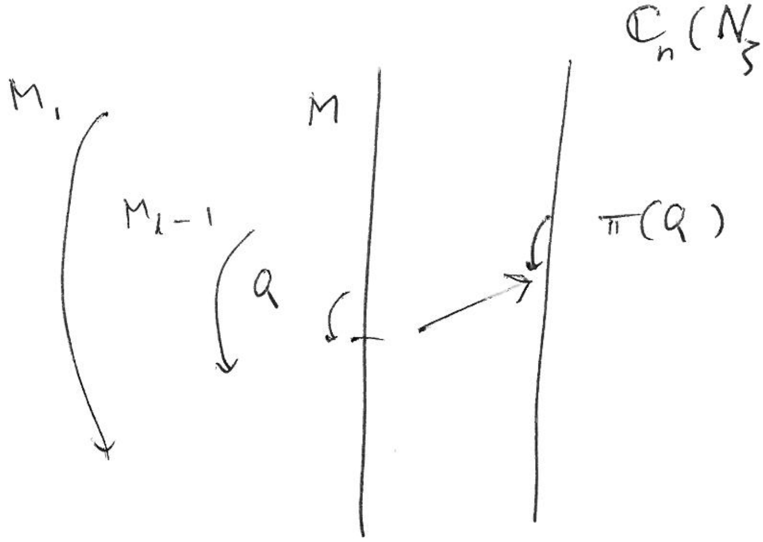
let $\gamma =$ height of stack

$N_\alpha = N_\gamma | \alpha$.

<u>resurrection.</u> given $\pi : M \longrightarrow \mathbb{C}_n(N_\zeta)$

(background Y) π weak n-embedding

given $Q \trianglelefteq M$, define $res^Y(\pi, \zeta, Q)$.



stan: $t_{\zeta_0} \neq 2$.

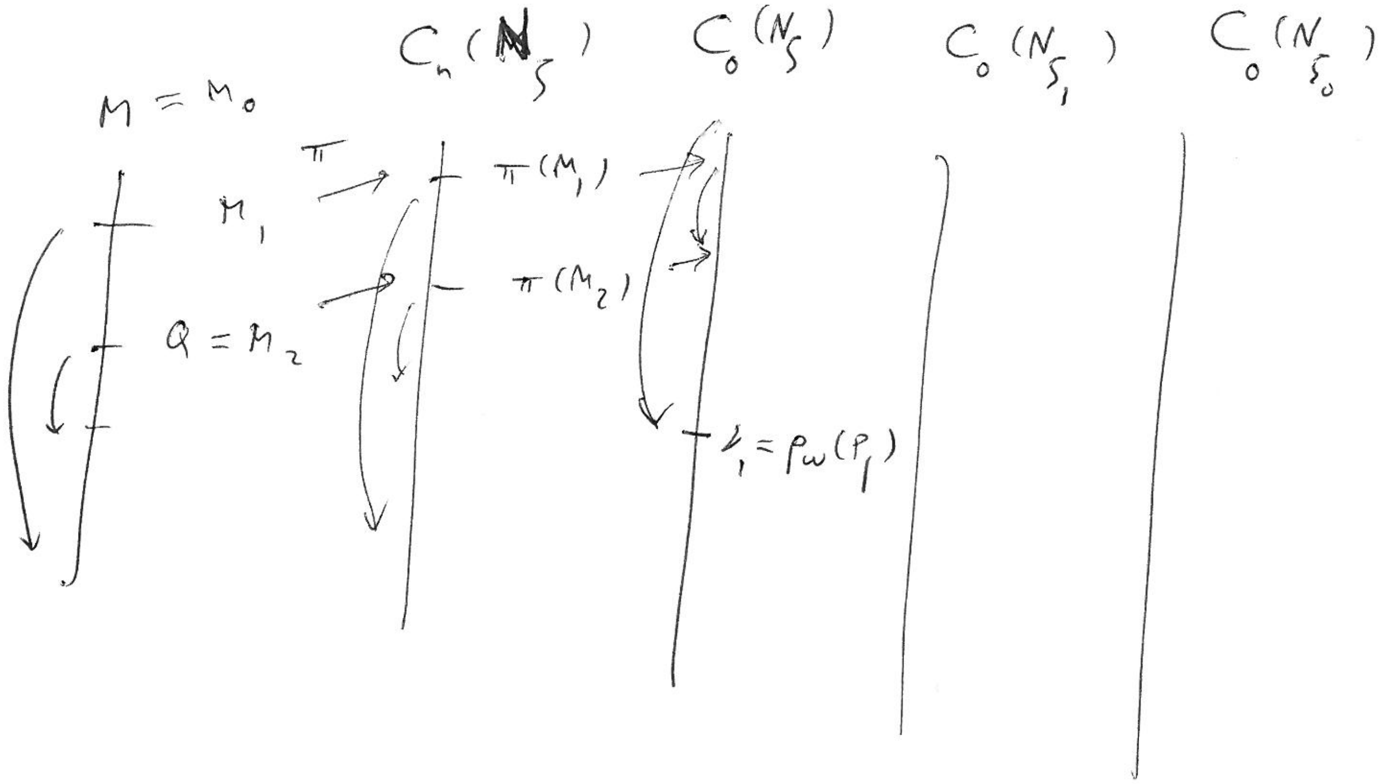
dropdown $(M, Q) = (M_i)_{i \le k}$

$Q = M_k$,

given M_i, let N least s.t.

$M_i \lhd N \lhd M$, $\rho_\omega(N) < \rho_\omega(M_i)$,
or $N = M$.

$\Rightarrow$ $\rho_\omega(M_1)$ is a cardinal in M



let $\zeta_1' =$ unique ζ s.t. $C_\omega(N_\zeta) = P_1$

$(\zeta_1' < \zeta_0)$

<u>case 1.</u> $t_{\zeta_1'} \neq 2 \Rightarrow E_0^{\mathcal{U}} = \emptyset$. $M_0^{\mathcal{U}} = M_1^{\mathcal{U}'}$.

write $\mathcal{U} = \text{restree}^Y(\pi, \zeta_0, Q)$

padded tree of length 3 on Y.

$\zeta_1 = \zeta_1'$.

$\pi_1 : M_1 \longrightarrow P_1$; $\pi_1 = \pi_{0,0} \restriction M_1$.

$\pi_{1,n} : M_1 \longrightarrow C_n(N_{\zeta_1})$

$P_2 = \pi_{1,n}(M_2)$, etc.

can ? $\quad t_{\zeta_2'} = 2$.

$E_0^{\mathcal{U}}$ = order 0 measure on ν_1.

$\pi_1 : M_1 \to P_1$ is $\pi_{0,0} \upharpoonright \mu_1$.

$M_1^{\mathcal{U}} \models$ "ζ_1 = unique $\bar{\zeta}$ s.t. $C_\omega(\bar{\zeta}) = \rho_1$".

⋮

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for Π^1_3.

φ a Π^1_3 fmla. $\quad A_\varphi = \{x : \varphi(x)\}$

$\varphi(x) \equiv \forall y\, \bar{\varphi}(x, y)$

a φ witness is an iterable x-mouse N,

$N \models \delta$ woodin

$N \models$ forced in ext algebra at δ

than $\bar{\varphi}(x, y)$ holds for y

the generic

$N \models$ a bit of thy.

assume $M_1^{\#}(x)$, all x.

$\varphi(x) \iff \exists\,\varphi$ witness for x

M_x = least φ-witness for x.

norms of scale compare properties of M_x, M_y for $x, y \in A_\varphi$.

$x \leq_n y \iff$ f.a./some mouse P s.t. $\mathbb{C}^P_x$ reaches M_x or $\mathbb{C}^P_y$ reaches M_y, then $x \leq^P_n y$

lifting norm

P's version of $\leq_n$.

$x \leq^P_t y$ iff ...

first, let Ω_x = unique Ω s.t. $\mathbb{C}_\omega(N^P_{x\,\Omega}) = M_x$
$= \infty$ if $\neg\exists$

$\Omega_y = \ldots\ldots$

if $\Omega_x < \Omega_y$, then $x <^P_t y$.

if $\Omega_x = \Omega_y$, then $x \leq^P_t y$ iff

$$t^{N^P_{\Omega_x}} x \leq t^{N^P_{\Omega_y}} y.$$

<u>iterability.</u>

idea: fix e, name for an extender, $E \in \mathbb{E}$

a, name for gen. of that ext $\alpha \subset \iota(E)$, given M_x, let $E = e^{M_x} \in \mathbb{E}^{M_x}$

$$\alpha = a^{M_x} \in \mathcal{L}(E).$$

let $\mathcal{T}$ = a tree on M_x, $\mathrm{lh}(\mathcal{T}) = 2$.

$$E_0^{\mathcal{T}} = E$$

$$\pi_0 : M_x \longrightarrow N_{\Omega_x}$$

let $E^* =$ resurrection of E

let $\pi_1 : \mathrm{ult}_0(M_x; E) \longrightarrow i^P_{E^*}(N_{\Omega_x}\, x)$

$$\begin{array}{ccccc} t^{M_1^{\mathcal{T}}}(\alpha) \in & M_1^{\mathcal{T}} & \xrightarrow{\pi_1} & \cancel{\ldots} & \mathrm{ult}(P; E^*) \ni \pi_1(t^{M_1^{\mathcal{T}}}(\alpha)) \\ & \uparrow & & \uparrow & \uparrow \\ & M_0^{\mathcal{T}} = M_x & \xrightarrow[\pi_0]{} & N_{\Omega_x\, x} & P \end{array}$$

same for y.

res. to backgr. extender E_y

$$e^{M_y} = E_0^{\mathcal{T}_y}, \quad a^{M_y} = \alpha$$

now: lex. order

$$\mathrm{lh}(E_x^*), \quad \pi_1^x(t^{M_1^{\mathcal{T}_x}}(\alpha)) \qquad \text{vs.}$$

$$\mathrm{lh}(E_y^*), \quad \pi_1^y(t^{M_1^{\mathcal{T}_y}}(\alpha))$$

norm description.

tuple

$\rho^{M_x}_{n_0+1} = \omega < \rho^{M_x}_{n_0}$

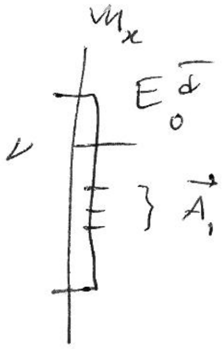
$e_i, \vec{a}_i$ are degree n_i+1 terms

$$\sigma = \left(({}_{n_0}e_0, \vec{a}_0), ({}_{n_1}e_1, \vec{a}_1), \ldots, ({}_{n_m}e_m, \vec{a}_m), t \right)$$

defn for M_x tree of lgth $\leq m+1$

$E_0^{I} = e_0^{M_x}$, $\quad \vec{A}_1 = \vec{a}_0^{\,M_x}$

(if $e_0^{M_x}$ undef. or $\notin \mathbb{E}_+^{M_x}$, then stop)



$E_1^{I} = e_1^{M_1^{I}}(\vec{A}_1)$, $\quad \vec{A}_2 = \vec{A}_1 {}^\frown \vec{a}_1(\vec{A}_1)$.

given x, y have $I_x^{\sigma}, I_y^{\sigma}$.

P reaches M_x or M_y.

defn $\leq_{\sigma}^{P}$: lex order of tq.

$$(\Omega_x^P, \underbrace{pars_0, pars_1, \text{—}, pars_m, \alpha_t})$$

C_x^P reaches M_x $\qquad$ if it does reach M_x

let $\mathcal{U}^{I_x}$ = lift/resurrect tree on P given by I_x

$pars_i = (\ldots)$ seq. of ordinals for resurrection (E_i^{I})

pars$_i$

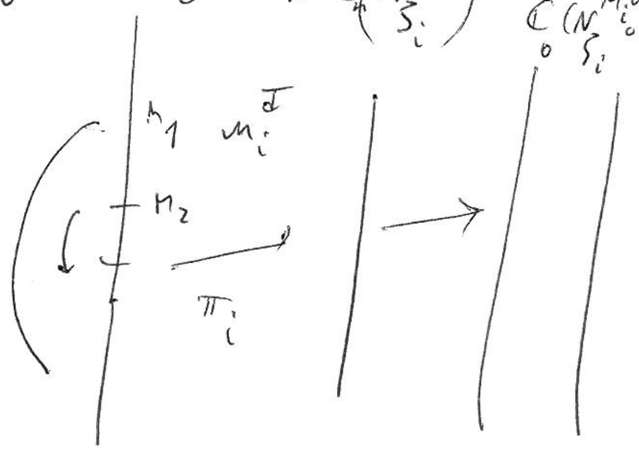
stage 2i $\rightarrow$ $\begin{cases} 0 & \text{if } e_i^{M_i^{\mathcal{I}}}(\vec{A}_i),\ \vec{a}_i^{M_i^{\mathcal{I}}}(\vec{A}_i) \text{ are sensible} \\ 1 & \text{o.w.} \rightarrow \text{stop} \end{cases}$

stage 3i $\rightarrow$ $k_i^{\mathcal{I}}$ { dropdown$(M_i^{\mathcal{I}}, E_i^{\mathcal{I}})$ has length $k_i^{\mathcal{I}}+1$

stage 4i $\rightarrow$ $\text{pars}_{i0} {}^\frown \cdots {}^\frown \text{pars}_{ik}$

$M_0 = \mathcal{I} = \mathcal{I}_x\ \mathbb{C}_*(N_{\zeta_i}^{M_{i_0}^{\mathcal{U}}})$ $\quad$ $\mathbb{C}_0(N_{\zeta_i}^{M_{i_0}^{\mathcal{U}}})$

$\text{pars}_{ij} = (\zeta_{ij}, t_{\zeta_{ij}}, p_{\zeta_{ij}}, \nu_{i,j+1}, N_{\zeta_i'1}, \zeta'_{i,j+1}, t_{\zeta'_{i,j+1}}, p_{\zeta'_{i,j+1}})$.



given background Y, $\quad p_\zeta^Y$ = ordinal height of production steps for stage ζ.

$p_\zeta = \text{lh}(E_\zeta^{\mathbb{C}})$ when $t_\zeta = 1$.

(when $i=m$ stage 5i) $\quad \pi_m(t^{M_m^{\mathcal{I}}}(\vec{A}_m))$

stage 5i, $i<m$ $\quad \begin{cases} 0 & \text{if } n_{i+1} = \deg^{\mathcal{I}}(i+1) \\ 1 & \text{o.w.} \rightarrow \text{stop.} \end{cases}$

<u>issues</u> :

- $\leq^P_\sigma$ is indep. of P
  $\Rightarrow$ it's a pwo.

- M_∞ is iterable.
  hence M_∞ is ω-iterable.

  <u>pf.</u>: fix finite tree $\mathcal{T}$ on M_∞
  fix σ s.t. $\mathcal{T} = \mathcal{T}^\sigma_{M_\infty}$.

fix mouse P s.t.

$x, (x_n) \in P$, and $\mathbb{C}^P_{x_n}$ reaches M_{x_n}.

defn $\mathcal{T}_{x_n} = \mathcal{T}^\sigma_{M_{x_n}}$ for large n

($lh(\mathcal{T}_{x_n}) = lh(\mathcal{T})$.

$M_i^{\mathcal{T}} \ni \alpha$

pick t s.t. $\alpha = t^{M_i^{\mathcal{T}}}(\vec{A}_0)$ e.g.

h $\mathcal{U}_{x_n}$ = lift of $\mathcal{T}_{x_n}$ to P

<u>claim</u>. $\mathcal{U} = \mathcal{U}_{x_n}$ is constant for large n
(as prod. of !'s are included.)

define $\pi_i : M_i^{\bar{d}} \cap OR \longrightarrow OR^{M_{i_0}^{\mathcal{U}}}$

$$\pi_i(\alpha) = \lim_{n \to \omega} \pi_{x_{n,i}}(\alpha_n) .$$

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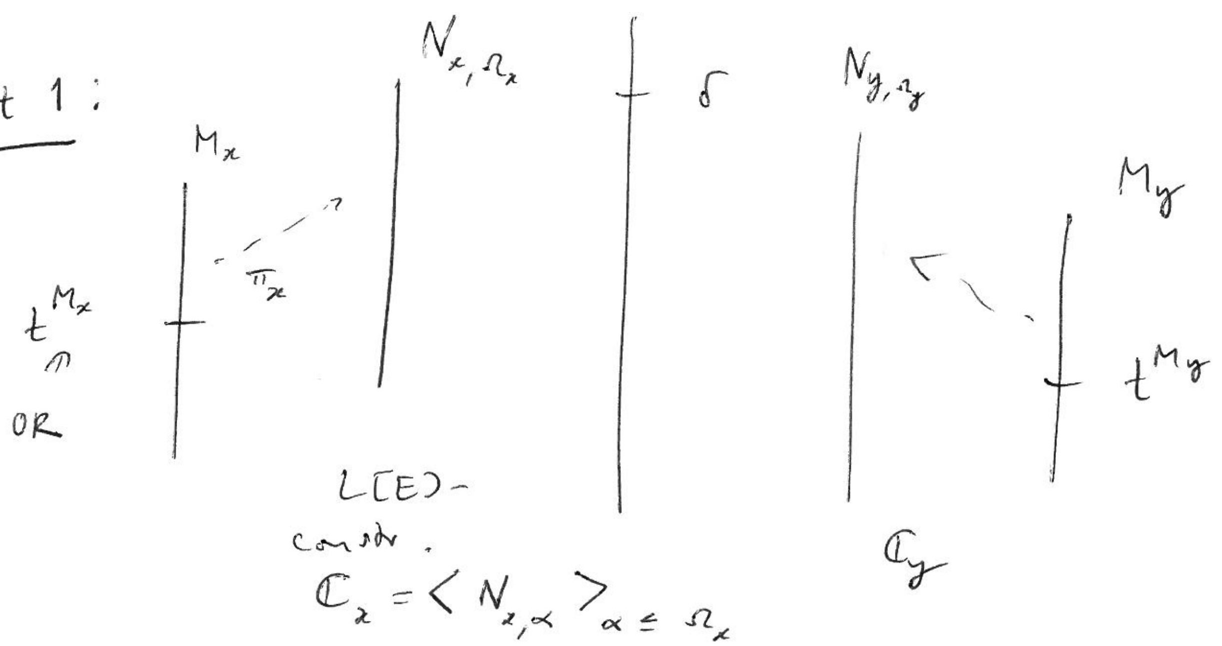
$A_\varphi = \{ x : M_x \models \varphi \}$ $\qquad x \mapsto M_x$ mouse operator

$\qquad\qquad '' \qquad\qquad\qquad\qquad\qquad \uparrow$

$\{ x : \varphi(x) \}$ $\qquad$ min. Π^1_3 witness to φ

dept 1:



1st norm: $x \le_0 y$ iff $\Omega_x \le \Omega_y$.

assume $\Omega_x = \Omega_y$.

$\pi_x : M_x \longrightarrow N_{x,\Omega_x}$ core embedding.

$x \le_t y$ iff $\Omega_x \le \Omega_y$ or

$$\pi_x(t^{M_x}) \le \pi_y(t^{M_y}).$$

with a mouse P replacing V, $x \le^P_t y$ iff

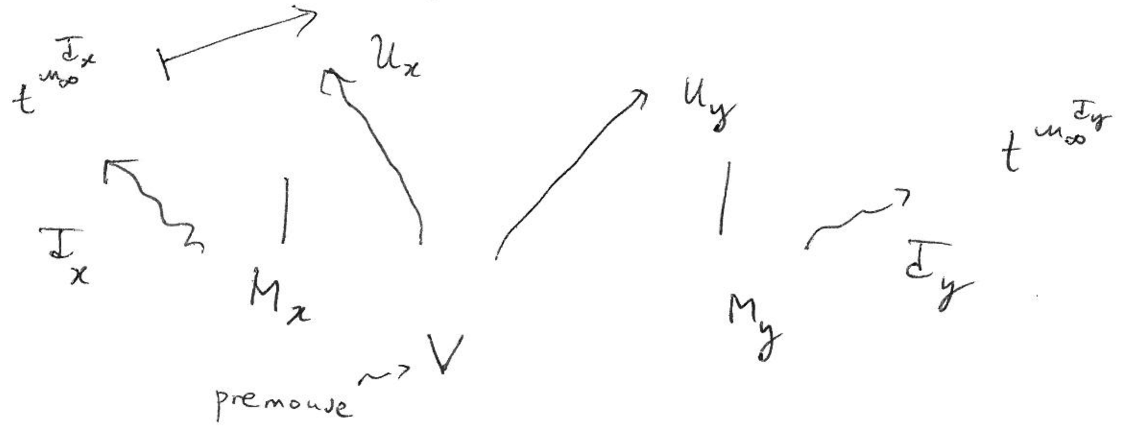
$P \models$ "$\Omega_x < \Omega_y$ or $\pi_x(t^{M_x}) \le \pi_y(t^{M_y})$"

need: $\leq^P_t$ independent of suff. mouse P.

<u>depth:</u> code τ for length $m+1$ it. tree

$\downarrow$

seq. of terms $t^{N_{x,\alpha}^{\mathcal{M}_\infty^{\mathcal{U}_x}}}$ $\quad t^{N_{y,\beta}^{\mathcal{M}_\infty^{\mathcal{U}_y}}}$



norm $\leq_\sigma$ compares x, y for features relating to $\mathcal{U}_x$, $\mathcal{U}_y$, including extenders $\mathcal{U}_x$, $\mathcal{U}_y$.

if $\mathcal{U}_x \neq \mathcal{U}_y$, we're done.

supp. $\mathcal{U}_x = \mathcal{U}_y$.

compare α vs. β.

if $\alpha = \beta$: compare $\pi_\infty^{\mathcal{I}_x}\left(t^{\mathcal{M}_\infty^{\mathcal{I}_x}}\right)$ vs.

$$\pi_\infty^{\mathcal{I}_y}\left(t^{\mathcal{M}_\infty^{\mathcal{I}_y}}\right).$$

iterability of $\mathcal{M}_\infty$:

def. an (m,λ)-(abstract) ~~at~~ lifting algorithm for

(M, P) is a pair (Σ, Ψ) s.t.

premouse

m-sound premouse

- Σ is an (m,λ)–strategy for M
- $\Psi = \left\langle \mathcal{U}^{\mathcal{T}}, \langle \pi^{\mathcal{T}}_\alpha, \zeta^{\mathcal{T}}_\alpha, e^{\mathcal{T}}_\alpha, \eta^{\mathcal{T}}_\alpha \rangle_{\alpha < lh(\mathcal{T})} \right\rangle_{\mathcal{T} \in X}$

X = set of trees via Σ (in part, of length $< \lambda$)

$\mathcal{U}^{\mathcal{T}}$ = "lift" of $\mathcal{T}$ to tree on P, same tree order as $\mathcal{T}$.

$\pi^{\mathcal{T}}_\alpha : OR \cap \mathcal{M}^{\mathcal{T}}_\alpha \longrightarrow \eta^{\mathcal{T}}_\alpha \in OR^{\mathcal{M}^{\mathcal{U}}_\alpha}$.

$\zeta^{\mathcal{T}}_\alpha \in OR \cap \mathcal{M}^{\mathcal{U}}_\alpha$.

$$M^{\mathcal{T}}_\alpha \xrightarrow{\tilde{\pi}^{\mathcal{T}}_\alpha} N^{M^{\mathcal{U}}_\alpha}_{\zeta^{\mathcal{T}}_\alpha} \qquad \pi^{\mathcal{T}}_\alpha = \tilde{\pi}^{\mathcal{T}}_\alpha \restriction OR$$

$\mathcal{U}$ = lift

$M^{\mathcal{T}}_\beta$ $\quad \mathcal{T}$ $\quad N^{M^{\mathcal{U}}_\beta}_{\zeta^{\mathcal{T}}_\beta}$

P

M

$$e_\alpha^{\mathcal{I}} : \{ N : N \trianglelefteq M_\alpha^{\mathcal{I}} + N \text{ active} \} \longrightarrow$$
$$\{ \text{extenders of } M_\alpha^{\mathcal{U}} \}$$

now list all commutativity prop's from resurrection/lifting.

<u>lem.</u> P iterable + $\exists (m,\omega)$-lifting algo $\mathcal{A}$ for (M, P) $\Rightarrow$ $\exists!$ extension of $\mathcal{A}$ to (m, λ) — lifting algo for (M, P).

---

have $\leq_\sigma$ def. for non descript'n σ.

<u>clai.</u> $\leq_\sigma$ is a pwo.

pf.: suff. to see $\forall x, y \in \mathbb{R}$ $\forall$ mice P, Q
$x \leq_\sigma^P y$ iff $x \leq_\sigma^Q y$.

(then given X ctbl set of reals, let P be a mouse with $X \subset P$.
consider $\leq_\sigma^P$ $\longrightarrow$ a pwo on $\mathbb{R}^P$)

$$\mathcal{I}_x = \mathcal{I}_x^{\sigma} = \mathcal{I}_{\sigma}^{M_x} .$$

need $\Omega_x^P \leq \Omega_y^P \iff \Omega_x^Q \leq \Omega_y^Q$

$$\begin{tikzcd}
 & P' & & & Q' & \\
 & P \arrow[u] & & & Q \arrow[u] & N^{Q}_{x,\Omega_x} \\
 & W^{P}_{x,\Omega_x} & W^{P}_{y,\Omega_y} & & N^{Q}_{y,\Omega_y} & \\
M_x \arrow[ur] & & & M_y \arrow[ul] \arrow[ur] & & M_x \arrow[uu]
\end{tikzcd}$$

$\mathbb{C}_x^{P'} = \mathbb{C}_x^{P'}$ up thru their common length.

same for y

if $\Omega_x^{P'} < \Omega_y^{P'}$, then same for P, likewise for Q'.

$$\begin{array}{cc} \Omega_x^{P'} & \Omega_y^{P'} \\ \parallel & \parallel \\ \Omega_x^{Q'} & \Omega_y^{Q'} \end{array}$$

similar for the interpretation of the terms.

to get $p \xrightarrow{u} p'$, $Q \xrightarrow{v} Q'$:

E_0^u, E_0^v :

let $\xi_{x,0}$ = least ξ s.t. P, Q are ξ-incompatible :

$F_\xi^{\mathbb{C}_x^P} \restriction X \neq F_\xi^{\mathbb{C}_x^Q} \restriction X$, where

$X = (P \cap Q) \times [\lambda]^{<\omega}$, $\lambda = \min(\mathcal{U}(F_\xi^{\mathbb{C}_x^P}), \mathcal{U}(F_\xi^{\mathbb{C}_x^Q}))$

⋮

some for E_α^u, E_α^v.

<u>claim 1.</u> $E_\alpha^u \perp E_\alpha^v$

<u>claim 2.</u> $\alpha < \beta \implies \xi_\alpha < \xi_\beta$

<u>claim 3.</u> no $t = 3$ background extenders used (from P-constr.)

<u>claim 4.</u> $\alpha < \beta \implies$

$\mathbb{C}^{m_\alpha^u} \restriction \xi_\alpha = \mathbb{C}^{m_\beta^u} \restriction \xi_\alpha$.